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Analyticity of non-stationary processes of change in diagnostic parameters of hydrostatic transmissions of harvesters

Abstract. The creation of competitive agricultural machinery on the world market is an urgent task. And the solution of this problem is primarily related to the issues of improving reliability. This is especially relevant for complex and therefore expensive agricultural machinery. It becomes an unacceptable luxury to operate combine harvesters where 50% of all working time is spent on downtime. These downtimes are associated with insufficient technical level and low reliability. Especially noteworthy is the low reliability of the drive elements. One of the ways to increase the energy saturation of complex agricultural machines is to replace mechanical gears for driving working bodies with hydraulic ones. The increase in the initial cost of the machine due to this can be compensated by reducing the cost of spare parts in further operation and reducing downtime. In the designs of Ukrainian and foreign manufacturers of agricultural machinery, a promising area today is the creation of a multifunctional hydro-mechanical drive. The purpose of the study is to confirm the existence of analytical approaches to describing non-stationary processes of changing the diagnostic parameters of hydrostatic transmissions of combine harvesters. The methodology is based on the Lagrange principle, developed elements of the fundamental provisions of the mechanics of continuous media with movable boundaries in relation to hydraulic drive systems of hydrostatic transmissions of combine harvesters, which allow expanding the field of research and modelling of diagnostics of these systems. The paper proves that in the field under study, the Bless method is a more effective method for numerically solving such systems of equations. It is shown that for one-dimensional motions of incompressible liquid media moving in the channel and bounded by mobile boundaries, the calculation is reduced to solving the equation $a(x, t)\,\ddot{x}=b(x, t)\,\dot{x}^2+c(x, t)$. Where $x=x(t)$ is the coordinate of the anterior or posterior boundary of the liquid medium moving in the channel. The authors of this study prove that this equation is a generalised Bernoulli equation for the case of motion of incompressible liquid media with moving boundaries. The proposed method allows calculating the dynamics of starting an ampulised hydraulic drive system for hydrostatic transmissions of combine harvesters with minimal volumes of 1...10% of gas cavities for storing the drive working fluid. The elements of the theory described above and the developed calculation methods allow expanding the field of research of dynamic modes of operation of hydraulic systems of the power drive of hydrostatic transmissions of combine harvesters in the process of filling the working fluid of hydraulic system channels with branches and hydraulic supports

Keywords: methodology, transmission, hydrostatics, efficiency, hydraulic systems

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INTRODUCTION

The operation of hydraulic systems of hydrostatic transmissions of combine harvesters in severe operating conditions is accompanied by complex high-speed hydrodynamic processes caused by the movement of liquid and gas-liquid media in the hydraulic channels and working cavities of hydraulic machines of these systems [1]. One of the most complex phenomena is the processes caused by the movement of these media, which have mobile boundaries [2]. These phenomena are mainly associated with the removal and filling of hydraulic channels and working cavities of hydraulic drive systems of hydrostatic transmissions of combine harvesters with working fluid [3]. They

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are typical for dynamic modes of operation of hydraulic systems: during overloads, during transient modes, during adjustment, refuelling, starting, and stopping [4]. They constantly accompany the operation of volumetric hydraulic units of hydraulic drive systems of hydrostatic transmissions of combine harvesters [5]. In general, these processes are typical for the operation of hydraulic systems of hydrostatic transmissions of combine harvesters operating on the principle of displacement of working fluids [6].

Complex problems of hydromechanics of liquid media with moving boundaries arise when studying the phenomena of not complete, but partial filling with working fluid of the cavities of hydraulic cylinders of volumetric units of hydraulic systems of hydrostatic transmissions of combine harvesters [7]. These phenomena take place during cavitation, which occurs due to a decrease in fluid pressure anywhere in the channel [8]. They can appear, for example, when the gas pressure drops in the gas cavities of tanks for storing working fluids of hydraulic systems [9]. They occur when starting the hydraulic pump drive or in transient modes of their operation, when the speed of rotation of the pump shaft exceeds the calculated values [10]. In this case, the liquid does not have time to fill the channels of the hydraulic cylinders due to the high speed of rotation of the cylinder block or the low pressure at the pump inlet [11].

The considered processes of hydraulic systems of drives of hydrostatic transmissions of combine harvesters are characterised by a sharp release of gases dissolved in liquid, flow continuity breaks, they are accompanied by the phenomena of incomplete water hammer, flow separation from the walls of the main channels and their incomplete filling [12].

In general, the study of the processes of motion of liquid and gas-liquid media with moving interface of media such as: "liquid-gas" or "liquid-solid", "liquid-piston" refers to the tasks of hydromechanics with contact breaks of media [13]. The main content of these tasks is to determine the laws of motion of mobile boundaries of discontinuities [14]. After that, the calculation of other flow parameters does not cause significant difficulties [15]. As already noted [16], the phenomena and processes described above significantly affect the technical condition of hydrostatic transmissions of combine harvesters, but they are poorly understood. This is conditioned by their physical complexity, difficulties in calculating and modelling them, and limited possibilities for observing the features of these phenomena and measuring their physical parameters [17].

In addition, the scope of setting and solving problems of hydromechanics with moving boundaries of media for hydraulic drive systems is limited by the Euler principle traditionally used in hydraulics [18]. Studies show that here, along with the Euler principle, it is necessary to apply the Lagrange principle, which is more general in physical and mathematical terms, since it allows setting and solving problems of hydromechanics with moving boundaries of the medium [19]. Its use would allow developing the fundamental provisions of the mechanics of continuous media with moving boundaries in relation to dynamic problems of changing the diagnostic parameters of hydrostatic transmissions of combine harvesters [20]. This would significantly expand the areas of research and solving problems of hydromechanics of changing the diagnostic parameters of hydrostatic transmissions of combine harvesters, improve the quality, accuracy and reliability of calculations, and the adequacy of modelling real processes that accompany their work [21].

The purpose of the study is to confirm the existence of analytical approaches to describing non-stationary processes of changing the diagnostic parameters of hydrostatic transmissions of combine harvesters.

RESULTS AND DISCUSSION

The study considers the problem of calculating the characteristics of processes of one-dimensional unsteady motion of liquid media with moving boundaries in channels of complex geometric shapes. It is assumed that the area $\sigma(s)$ of the channel cross-section is a given function of the curved coordinate s counted along the channel axis. A liquid medium moves in the channel, bounded by two moving surfaces and the side surface of the channel: the boundaries of the medium. The areas of moving surfaces are equal to $\sigma_1(t)=\sigma[s_1(t)]$ and $\sigma_2(t)=\sigma[s_2(t)]$, the area of the side surface of the channel is equal to $\sum(t)$ (Fig. 1).

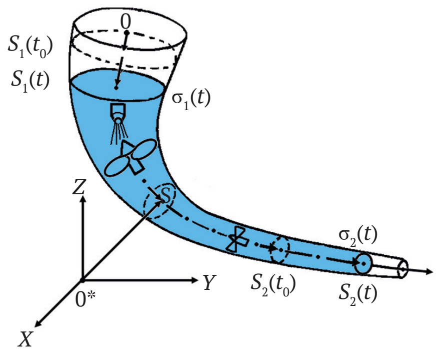

Figure 1. Movement of liquid media with movable boundaries in the channels of pipelines of hydraulic systems of drives of hydrostatic transmissions of combine harvesters

It is assumed that the pressure of a liquid medium on moving surfaces is given. Since the liquid medium considered here consists of the same liquid particles, and the boundary conditions of the problem are set at the moving boundaries of the medium, the Lagrange approach can be used as the most appropriate research principle in such cases.

The position of moving surfaces in the channel is determined by Euler curved coordinates $s_0(t)=s(\xi_1, t)$ and $s_2(t)=s(\xi_2, t)$, where ξ_1 and ξ_2 – Lagrange coordinates indicating the state of the surfaces under consideration at the initial time t=0. The movement of the liquid is carried out under the action of mass and surface forces. The main relations describing the patterns of movement of these surfaces in these channels are obtained. It is assumed that the pressure $p_1=p(s_1, t)$ and $p_2=p(s_2, t)$ on the surfaces of these boundaries are known functions of Euler coordinates s_1 and s_2, and time t.

The theory and calculation methods are based on general integral relations expressing the basic laws of mechanics of continuous media with moving boundaries (Fig. 1), namely, the laws of change: masses $m(t)$ of medium, its kinetic energy $E(t)$, the amount of movement $\bar{K}(t)$ of environment, and the moment $\bar{L}(t)$ of its quantities of motion are represented as equations:

$$\frac{dm}{dt} = \frac{d}{dt}\int_{V(t)} \rho dV = \int_{V(t)} \frac{d\rho}{dt} dV + \int_{\sigma_2(t)} \rho v_n d\sigma - \int_{\sigma_1(t)} \rho v_n d\sigma = \sum_{(i,j)} \dot{m}_{i,j} \quad (1)$$

$$\frac{dE}{dt} = \frac{d}{dt}\int_{V(t)} \rho \frac{1}{2} v^2 dV = \int_{V(t)} \frac{d}{d(t)} [\rho \frac{1}{2} v^2] dV + \int_{\sigma_2(t)} \rho \frac{1}{2} v^2 v_n d\sigma - \int_{\sigma_1(t)} \rho \frac{1}{2} v^2 v_n d\sigma = \int_{V(t)} \rho \cdot \bar{f} \cdot \bar{v} dV + \int_{\sigma_1(t)} \rho \cdot \overline{v_n} d\sigma - \int_{\sigma_2(t)} \rho \cdot \overline{v_n} d\sigma - \dot{D} + \sum_{(i,j)} \dot{E}_{i,j} \quad (2)$$

$$\frac{d\bar{K}}{dt} = \frac{d}{dt}\int_{V(t)} \rho\bar{v} dV = \int_{V(t)} \frac{d}{dt} (\rho\bar{v}) dV + \int_{\sigma_2(t)} (\rho\bar{v}) v_n \cdot d\sigma - \int_{\sigma_1(t)} (\rho\bar{v}) v_n \cdot d\sigma = \int_{V(t)} \bar{f}\rho dV + \int_{\sigma_1(t)} \bar{p}_n d\sigma - \int_{\sigma_2(t)} \bar{p}_n d\sigma - \int_{\Sigma(t)} \bar{\tau}_n d\sigma + \sum_{(i,j)} \dot{\bar{K}}_{i,j} \quad (3)$$

$$\frac{dL}{dt} = \frac{d}{dt}\int_{V(t)} (\bar{r} x \rho\bar{v}) dV = \int_{V(t)} \frac{d}{dt} (\bar{r} x \rho\bar{v}) dV + \int_{\sigma_2(t)} (\bar{r} x \rho\bar{v}) \bar{v}_n d\sigma - \int_{\sigma_1(t)} (\bar{r} x \rho\bar{v}) \bar{v}_n d\sigma = \int_{V(t)} (\bar{r} x \rho\bar{f}) dV + \int_{\sigma(t)} (\bar{r} x \bar{p}_n) d\sigma + \sum_{(i,j)} \dot{\bar{L}}_{i,j} \quad (4)$$

where: $\bar{v}$ – vector of the absolute velocity of movement of a given particle of the medium, m/s; ρ, kg/m^3 – density of the medium; $\bar{f}$, m/s^2 and $\bar{\rho}=\bar{\rho}_n+\bar{\rho}_\tau$, N/m^2 – respectively, vectors of stresses of mass and surface forces acting on the particle of the medium and its surface; $\sum_{(i,j)} \dot{m}_{i,j}$, kg/s, $\sum_{(i,j)} \dot{E}_{i,j}$, W – the sum of the capacities of additional sources and effluents of mass and mechanical energy, respectively, located inside the moving medium; $\sum_{(i,j)} \dot{\bar{K}}_{i,j}$, kg·M/s^2, $\sum_{(i,j)} \dot{\bar{L}}_{i,j}$, kg·m^2/s^2 – the sum of the capacities of additional sources and drains and the moment of the amount of movement, respectively, located inside the moving medium; $\bar{r}$ – radius-vector drawn to a given point of the medium from point O* – the centre of its rotation (Fig. 1).

Notably, the integration region is mobile and depends on time t, s. Integral relations (1)-(4) will be used in deriving equations describing the processes of motion of incompressible liquid media with moving boundaries in the channels of hydraulic drive systems by vane and volumetric hydraulic machines, and with jet devices.

In the one-dimensional case for movement in the channel of a compressed liquid medium bounded by mobile surfaces $\sigma_1(t)$ and $\sigma_2(t)$ (Fig. 1), the above relations will take the form:

$$\int_{S_1}^{S_2} \frac{d\rho(s,t)}{dt} \sigma(s) ds + \rho(s_{2,},t) v(s_{2,},t) \sigma(s_2) - \rho(s_{1,},t) v(s_{1,},t) \sigma(s_1) = \sum_{(i,j)} m_{i,j} \quad (5)$$

$$\int_{S_1(t)}^{S_2(t)} \frac{d}{dt} [\rho(s,t)\bar{v}(s,t)] \sigma(s) ds + \rho(s_2,t)\bar{v}(s_2,t) v(s_2(t)\sigma(s_2) - \rho(s_1,t)\bar{v}(s_1,t) v(s_1(t)\sigma(s_1) = \int_{S_1(t)}^{S_2(t)} \rho(s,t)\bar{f}(s,t)\sigma(s) ds + \int_{\sigma_1(t)} \bar{p}_n d\sigma - \pi \cdot \int_{S_1(t)}^{S_2(t)} \bar{\rho}_\tau \cdot \delta(s) ds + \sum_{(i,j)} \bar{K}_{i,j} \quad (6)$$

$$\int_{S_1(t)}^{S_2(t)} \frac{d}{dt} \left[\frac{1}{2}\rho(s,t) v^2(s,t)\right] \sigma(s) ds + \frac{1}{2}\rho(s_2,t) v^3(s_2,t)\sigma(s_2) - \frac{1}{2}\rho(s_1,t) v^3(s_1,t)\sigma(s_1) = \int_{S_1(t)}^{S_2(t)} \rho(s,t)\bar{f}(s,t)\bar{v}(s,t)\sigma(s) ds + p(s_1,t) v(s_1,t)\sigma(s_1) - p(s_2,t) v(s_2,t)\sigma(s_2) - \dot{D} + \sum_{(i,j)} N_{i,j} \quad (7)$$

$$\int_{S_1(t)}^{S_1(t)} \frac{d}{dt} [\bar{r}(s,t) \text{ x } \rho(s,t)\bar{v}(s,t)]\sigma(s) ds + [\bar{r}(s_2,t) \text{ x } \rho(s_2,t) \text{ x } \bar{v}(s_2,t)]\sigma(s_2) - [\bar{r}(s_1,t) \text{ x } \rho(s_1,t) \text{ x } \bar{v}(s_1,t)]\sigma(s_1) = \int_{S_1(t)}^{S_2(t)} [\bar{r} \text{ x } \rho(s,t)\bar{f}(s,t)]\sigma(s) ds + \int_{\sigma_1(t)} (\bar{r} \text{ x } \bar{p}) d\sigma - \int_{\sigma_2(t)} (\bar{r} \text{ x } \bar{p}) d\sigma + \sum_{(i,j)} L_{i,j} \quad (8)$$

where: $Y(s, t)$, m/s – the average absolute velocity of liquid particles in section s at time t; $p(s_1, t)$, $p(s_2, t)$, Pa –average pressure on moving surfaces; $\dot{D}$, W – the rate of dissipation of mechanical energy of the medium due to friction and eddy formation, equal to:

$$\dot{D} = \frac{1}{2}\lambda Q^3(t)\rho \int_{S_1(t)}^{S_2(t)} \frac{ds}{\sigma^2(s)\delta(s)} \text{sign } v \quad (9)$$

where: λ – coefficient of fluid energy loss in the Darcy-Weisbach equation; δ(s) – channel diameter, m.

Equation (9) can be used for all currents satisfying the condition $\lambda=\lambda(Re,\Delta_{III}/\delta)$, where Re – the Reynolds criterion, ΔW, m – height of the roughness bumps of the channel walls.

Notably, the form of writing conservation laws in the above form (5)-(8) is general, both in the Euler approach and in the Lagrange approach. The difference is as follows.

If the liquid medium moving in the channel has movable boundaries with coordinates $s=s_1(t)$ and $s=s_2(t)$, and consists of the same particles (Lagrange approach), then in this case $v(s_1 t) = \frac{ds_1}{dt}, v(s_2 t) = \frac{ds_2}{dt}$. If the area of the channel through which the liquid moves is bounded by fixed boundaries with coordinates $s_1=const=l_1$ and $s_2=const=l_2$ (Euler approach), then $v(s_1, (t)=v(l_1, t)$ and $v(s_2, (t)=v(l_2, t)$.

This form of notation is also valid in the case of the movement of a liquid medium bounded on one side by a stationary surface with a coordinate $s_1=const=l_1(v(s_1, t)=v_1(l_1, t))$, and on the other side – a movable surface with a coordinate $s_2(t), \left(v(s_2,t) = \frac{ds_2}{dt}\right)$. This case corresponds to filling in the channel.

This recording form is also valid if the channel is empty. In this case, the liquid medium is bounded, on the one hand, by a mobile surface with a coordinate $s_1(t)$, ($v(s_1, t)=\frac{ds_1}{dt}$), and on the other – a fixed surface with coordinates $const=l_2(v(s_2, t)=v_2(l_2, t))$.

For one-dimensional problems with moving boundaries of media $s_1(t)$ and $s_2(t)$ the relations for conservation laws, written as (6)-(8), can be obtained by formal time differentiation of the values $m(t)$ of the mass of a liquid, its kinetic energy $E(t)$, and the amount of motion $\bar{K}(t)$ of liquid, and the moment $\bar{L}(t)$ of its number of movements, represented, respectively, in the form of:

$$m(t) = \int_{S_1(t)}^{S_1(t)} \rho(s,t)\sigma(s) ds \quad (10)$$

$$E(t) = \frac{1}{2}\int_{S_1(t)}^{S_1(t)} \rho(s,t) v^2(s,t)\sigma(s) ds \quad (11)$$

$$\bar{K}(t) = \int_{s_1(t)}^{s_1(t)} \rho(s,t)\bar{v}(s,t)\sigma(s)ds \quad (12)$$

$$\bar{L}(t) = \int_{s_1(t)}^{s_1(t)} [\bar{r}(s,t) \text{x} \, \rho(s,t)\bar{v}(s,t)]\sigma(s)ds \quad (13)$$

Here the property of the derivative of the integral is used, which depends on the parameter t, when the integrand and the integration limits depend on this parameter:

$$\frac{d}{dt}\int_{s_1(t)}^{s_2(t)} f(s,t)ds = \int_{s_1(t)}^{s_2(t)} \frac{df(s,t)}{dt}ds + \frac{ds_2(t)}{dt}f(s_2(t),t) - \frac{ds_1(t)}{dt}f(s_1(t),t) \quad (14)$$

For incompressible liquid media, the law of conservation and energy conversion, considering the viscosity of the medium and the mobility of its boundaries, can be written in the form:

$$\frac{1}{2}\rho\int_{s_1(t)}^{s_2(t)} \frac{dv^2(s,t)}{dt}\sigma(s)ds + \frac{1}{2}\rho[v^2(s_2,t)\sigma(s)v(s_2,t) - v^2(s_1,t)\sigma(s_1)v(s_1t)] = \rho\int_{s_1(t)}^{s_2(t)}[\cos(\bar{v},\bar{X}) \cdot f_x(t) + \cos(\bar{v},\bar{Y})\cdot f_Y(t) + \cos(\bar{v},\bar{Z})\cdot f_z(t)]v(s,t)\sigma(s)ds + p(s_1,t)v(s_1,t)\sigma(s_1) - p(s_2,t)v(s_2,t)\sigma(s_2) + \sum_{(i,j)} N_{i,j} - \frac{1}{2}\lambda\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma^2(s)\delta(s)}Q^3(t)signv \quad (15)$$

For this case, the relations are valid:

$$\int_{s_1(t)}^{s_2(t)}\frac{dv^2(s,t)}{dt}\sigma(s)ds = \int_{s_1(t)}^{s_2(t)}\frac{d}{dt}\left[\frac{Q^2(t)}{\sigma(s)}\right]ds = \frac{dQ^2(t)}{dt}\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)} \quad (16)$$

Provided that mass forces manifest themselves only in the form of gravity forces:

$$f_x = f_y = 0, f_z = -g \quad (17)$$

$$\int_{s_1(t)}^{s_2(t)}[\cos(\bar{v},\bar{Z})\cdot f_z]\cdot v(s,t)\sigma(s)ds = g[Z(s_1) - Z(s_2)]Q(t) \quad (18)$$

where $[Z(s_1)-Z(s_2)]$ – the difference in the heights of the centres of the surfaces of movable borders above an arbitrary horizontal plane (Fig. 1). Considering the above ratios:

$$\frac{1}{2}\rho\frac{dQ^2}{dt}\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)} + \frac{1}{2}\rho\left[\frac{1}{\sigma^2(s_2)} - \frac{1}{\sigma^2(s_1)}\right]\cdot Q^3(t) = [p(s_1,t) - p(s_2,t)]\cdot Q(t) + g\rho[Z(s_1) - Z(s_2)]\cdot Q(t) - \frac{1}{2}\rho\left[\lambda\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma^2(s)\sigma(s)} + \sum_{(k)}\frac{\zeta_k}{\sigma_k^2}\right]\cdot Q^3(t) + \sum_{(i,j)} N_{i,j} \quad (19)$$

where ζ_k – in general, the time-dependent coefficient ofk-th local hydraulic resistance located at some point in the channel. For $Q(t)\neq0$:

$$\rho\frac{dQ}{dt}\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)} + \frac{1}{2}\rho\left[\frac{1}{\sigma^2(s_2)} - \frac{1}{\sigma^2(s_2)}\right]\cdot Q^2(t) = p(s_1,t) - p(s_2,t) + g\rho[Z(s_1) - Z(s_2)] - \frac{1}{2}\rho\left[\lambda\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma^2(s)\sigma(s)} + \sum_{(k)}\frac{\zeta_k}{\sigma_k^2}\right]\cdot Q^2(t) + \sum_{(i,j)}\Delta p_{i,j} \quad (20)$$

where $\sum_{(i,j)}\Delta p_{i,j} = \frac{1}{Q}\sum_{(i,j)} N_{i,j}$.

Counting (for certainty) in this equation $Q = \frac{ds_2}{dt}\sigma(s_2)$ and given that:

$$\frac{d}{dt}\sigma(s_2) = \frac{d}{dt}\sigma[s_2(t)] = \sigma'(s_2)\frac{ds_2}{dt}, \sigma'(s_2) = \frac{d\sigma}{ds}, s = s_2 \quad (21)$$

and also that:

$$\frac{dQ}{dt} = \frac{d}{dt}\left[\frac{ds_2}{dt}\sigma(s_2)\right] = \frac{d^2s_2}{dt^2}\sigma(s_2) + \sigma'(s_2)\left(\frac{ds_2}{dt}\right)^2 \quad (22)$$

obtain:

$$\rho\sigma(s_2)\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)}\ddot{s}_2 + \frac{1}{2}\rho\sigma^2(s_2)\left[\frac{1}{\sigma^2(s_2)} - \left[\frac{1}{\sigma^2(s_1)} + 2\frac{\sigma'(s_2)}{\sigma^2(s_2)}\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)}\right]\right]\cdot \dot{s}_2^2 = p(s_1,t) - p(s_2,t) + \rho g[Z(s_1) - Z(s_2)] - \frac{1}{2}\sigma^2(s_2)\left[\lambda\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma^2(s)\sigma(s)} + \sum_{(k)}\frac{\zeta_k}{\sigma_k^2}\right]\cdot\dot{s}_2^2 sign\,\dot{s}_2 + \Delta P \quad (23)$$

If it is accepted that $Q(t) = \frac{ds_1}{dt}\sigma(s_1)$, then the equation written above will have the form:

$$\rho\sigma(s_1)\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)}\ddot{s}_1 + \frac{1}{2}\rho\sigma^2(s_1)\left[\frac{1}{\sigma^2(s_2)} - \frac{1}{\sigma^2(s_1)} + 2\frac{\sigma'(s_1)}{\sigma^2(s_1)}\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma(s)}\right]\cdot\dot{s}_1^2 = p(s_1,t) - p(s_2,t) + \rho g[Z(s_1) - Z(s_2)] - \frac{1}{2}\sigma^2(s_1)\left[\lambda\int_{s_1(t)}^{s_2(t)}\frac{ds}{\sigma^2(s)\sigma(s)} + \sum_{(k)}\frac{\zeta_k}{\sigma_k^2}\right]\cdot\dot{s}_1^2 sign\,\dot{s}_1 + \Delta P \quad (24)$$

From equation (1) for the law of conservation of mass of a moving medium, in the absence of sources and drains of mass inside the liquid:

$$\frac{ds_1}{dt}\sigma(s_1) = \frac{ds_2}{dt}\sigma(s_2) \quad (25)$$

In this case, the acceleration of the boundaries of the moving medium is related by the equations:

$$\frac{d^2s_1}{dt^2}\sigma(s_1) + \sigma'(s_1)\left(\frac{ds_1}{dt}\right)^2 = \frac{d^2s_2}{dt^2}\sigma(s_2) + \sigma'(s_2)\left(\frac{ds_2}{dt}\right)^2$$

Relations (11) and (12) allow switching in equation (10) to a single variable, for example, $x(t)=s_2(t)$. The coordinate of the movable border is indicated by a lowercase letter x, in contrast to the X-axis designation, which is capitalised in Figure 1.

It follows from (11) that $\sigma(s_1)ds_1 = \sigma(s_2)ds_2$, hence, after integrating the left side of this equation within the range of ζ_1 to s_1 and the right one – within the limits of up to ζ_2 to s_2, obtain:

$$f(s_1) = f(\zeta_1) + f(s_2) - f(\zeta_2), \quad (26)$$

where:

$$f(y) = \int\sigma(y)dy, \; y = s_1, \zeta_1, s_2, \zeta_2. \quad (27)$$

Equations (13), (14) determine the relationship between the coordinates s1 and s2 in the form of:

$$s_1 = s_1(s_2) = s_1(x) \quad (28)$$

or

$$s_2 = s_2(s_1) = s_2(x). \quad (29)$$

In case of a minor change in the area $\sigma'(s)$ of cross-section of the channel, components containing the derivative of the area along the coordinate, i.e., the value of $\sigma'(s)$, can be ignored. In the case of filling or emptying tanks with a spherical or conical surface, the value of $\sigma(s)$ should be considered.

Now equation (10) can be written for a single variable, for example, $x(t)=s_2(t)$, i.e., relative to the coordinate x anterior fluid boundary:

$$\rho\sigma(x)f_1(x)\ddot{x} + \frac{1}{2}\rho\left[1 - \frac{\sigma^2(x)}{\sigma^2[s_1(x)]} + 2f_1(x)\sigma'(x)\right]\dot{x}^2 = p[s_1(x),t] - p(x,t) + g\rho\{Z[s_1(x)] - Z(x)\} - \frac{1}{2}\rho\sigma^2(x)\left[\lambda f_2(x) + \sum\frac{I_k}{\sigma_k^2}\right]\dot{x}^2 + \Delta p \quad (30)$$

where:

$$f_1(x) = \int_{s_1(x)}^{x}\frac{ds}{\sigma(s)}, \; f_2(x) = \int_{s_1(x)}^{x}\frac{ds}{\sigma^2(s)\delta(s)}$$

$$f_1(x) = \int_{s_1(x)}^{x} \frac{ds}{\sigma(s)},\ f_2(x) = \int_{s_1(x)}^{x} \frac{ds}{\sigma^2(s)\delta(s)}. \qquad (31)$$

If equation (31) uses the independent variable $x(t)=s_1(t)$ using relation (30), which establishes the relation $s_2=s_2(s_1)=s_2(x)$, then the equation with respect to the coordinate x posterior fluid boundary is obtained:

$$\rho\sigma(x)k_1(x)\ddot{x} + \frac{1}{2}\rho\left[\frac{\sigma^2(x)}{\sigma^2[s_2(x)]} - 1 + 2k_1(x)\sigma'(x)\right] \dot{x}^2 = p(x,t) - p[s_2(x),t] + \rho g\{Z(x) - Z[s_2(x)]\} - \frac{1}{2}\rho\sigma^2(x)\left[\lambda k_2(x) + \sum_k \frac{\zeta_k}{\sigma_k^2}\right]\dot{x}^2 sign\,\dot{x} + \Delta p \qquad (32)$$

where:

$$k_1(x) = \int_x^{s_2(x)} \frac{ds}{\sigma(s)},\ \ k_2(x) = \int_x^{s_2(x)} \frac{ds}{\sigma^2(s)\delta(s)}. \qquad (33)$$

In the case when one of the intersections s_1 or s_2 is stationary (for example, when liquid is released through cross-section $s_2=l$ of channel), $x=s$ should be taken as the desired variable $x=s_1(t)$, that is, the coordinate of the posterior boundary. In this case, the equation (32) should use $s_2(x)=l$:

$$\rho\sigma(x)k_1(x,l)\ddot{x}+\frac{1}{2}\left[\frac{\sigma^2(x)}{\sigma^2(l)}-1+2\sigma'(x)k_1(x,l)\right] \dot{x}^2=p(x,t)-p(l,t)+\rho g[Z(x)-Z(l)]-\frac{1}{2}\rho\sigma^2(x) \left[\lambda k_2(x,l)+\sum\frac{\zeta_k}{\sigma_2^2}\right]\dot{x}^2\mathrm{sign}\dot{x}+\Delta p \qquad (34)$$

where:

$$k_1(x,l) = \int_x^{l} \frac{ds}{\sigma(s)},\ \ k_2(x,l) = \int_x^{l} \frac{ds}{\sigma^2(s)\delta(s)}. \qquad (35)$$

In this general form, equation (34) can be used to determine the function $p=p(l, t)$. This function can be used to calculate the time change in pressure at an arbitrary (stationary) point l of the channel filled with working fluid. To do this, the expression for $p=p(l, t)$ is used, defined by equation (30), substituting the value $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, found as a result of solving general equations, for example, (28) or (29).

Thus, the calculation of parameters that characterise the dynamics of motion of liquid media with moving boundaries in channels of complex geometric shapes can be reduced to solving the Cauchy problem for some ordinary nonlinear second-order differential equation with respect to the coordinate $x(t)$ one of the movable boundaries and allowed relative to the highest derivative. This equation can be reduced to the form:

$$\ddot{x}_i=f(x_i, \dot{x}_i, t) \qquad (36)$$

It can be shown that the calculation of parameters that determine the dynamics of motion of liquid media with moving boundaries in branched channels with the number of moving boundaries equal to n+1 for example, when filling hydraulic lines with working fluid, it is reduced to solving the Cauchy problem for systems n of ordinary second-order differential equations with respect to coordinates xi of movable borders: $\ddot{x}_i=f(x_i, \dot{x}_i, t)$ $i=1...n$.

The equations obtained here are generalisations of the Bernoulli equation for the case of motion of liquid media with moving boundaries in channels of complex geometric shapes. Here is an example of using the Lagrange approach to calculate the motion characteristics of liquid media with moving boundaries in spherical coordinates. This approach is applied to constructing a model of the dynamics of a gas bubble in a gas-liquid medium. It was shown above that the main dynamic characteristics of the processes of one-dimensional motion of incompressible liquid media with moving boundaries in the channels of hydraulic lines, elements, and aggregates of hydraulic drive systems are described by ordinary nonlinear second-order differential equations of the form (34).

This type of equation allows directly determining the coordinates of the moving boundaries of a liquid medium, for example, the liquid flow front, the boundaries of the medium with a moving piston of a hydraulic cylinder, and the position of the surface of any other moving element bordering the liquid. The appearance of these equations also allows using the efficient methods specifically adapted to solve them.

The equations used in this paper, however, also have a specific feature due to the presence of discontinuous coefficients in them. The presence of such coefficients is determined by the need to consider (in the calculation process) in these equations additional terms that appear or "disappear" from the composition of this equation at a certain time. In this system of equations, new equations may appear or "disappear". This can be caused by the opening of valves, abrupt changes in the cross-sectional areas of hydraulic distributor channels during adjustment, and during transient modes of operation of hydraulic systems. Additional equations appear, for example, when starting injectors, vane, or volumetric pumps, and in other similar cases, when calculations need to take into account new sources or drains of mass, energy, amount of movement, or moment. This also happens when branches from the main lines of hydraulic systems are filled with liquid, in cases where new local hydraulic supports appear.

These features make it difficult to use standard programmes for solving systems of ordinary differential equations, for example, in the currently widely used MathCad mathematical environment. This is explained by the following.

The presence of discontinuous coefficients in the right-hand sides of ordinary differential equations written in Cauchy form makes it inefficient to apply many well-known methods for numerical integration of these equations. Thus, the use of one-step Runge-Kutta methods implies sufficient smoothness of the right-hand sides of equations and their derivatives. In addition, in these methods, to calculate the desired functions in the next step, it is necessary to calculate the value of the right side of equation (20) several times. This significantly increases the calculation time, since the right-hand sides of the equations described by the processes under study usually have cumbersome expressions. The Euler method is applicable in this case, but requires small integration steps, while providing little accuracy. The so-called k-step methods, which include methods based on finite-difference schemes of the Milne method, are not convenient in these cases, since when the coefficients of these equations are abruptly changed, it is necessary to calculate the beginning of the integration table fairly accurately each time, while, as a rule, one-step methods of the Runge-Kutta type are used. Evidently, all this significantly complicates the algorithm and the numerical calculation process itself, while increasing the calculation time.

The requirements for the device of mathematical support of CAD hydraulic drive systems lead to the need to search for such methods of numerical integration of equations that would have sufficient simplicity and efficiency in solving the problems described above. It is known that in the formal approach to the question of numerical integration, the Cauchy problem for equations of order above the first and their systems is reduced to the Cauchy problem for systems of first-order equations. However, methods specifically adapted to solve higher-order equations often turn out to be more rational.

These problems have led to the need to look for new ways and methods for solving systems of differential equations with ordinary derivatives containing discontinuous coefficients. Studies have shown that the Bless method was quite effective here. Its advantages are as follows. Firstly, this method is specifically designed for solving second-order differential equations. Secondly, this method has a very high accuracy: its error is about $0(h^6)$. Thirdly, in this method, at each step of numerical integration, it is enough to calculate the right-hand side of the differential equation only once, and only at the fifth step is refinement performed with high accuracy.

The essence of this method is explained by the example of solving a system of equations of the form (20). The essence of the method is to calculate in the next step (at the point t_{e+1}) of the desired vector function $\overline{x}(t)$ and its derivatives $\dot{\overline{x}}(t)$ and $\ddot{\overline{x}}(t)$ using relations:

$$\bar{x}(t_{k+1}) \approx \bar{x}(t_k) + h\cdot\dot{\bar{x}}(t_k) + \frac{1}{2}h^2\cdot\ddot{\bar{x}}(t_k).$$

$$\bar{x}(t_{k+1}) \approx \bar{x}(t_k) + h\ddot{\bar{x}}(t_k).$$

$$\ddot{\bar{x}}(t_{k+1}) = \bar{f}\left(\bar{x}(t_{k+1}), \dot{\bar{x}}(t_{k+1}), t_{k+1}\right),$$

where $t_{k+1}=t_k+h$; h – integration step.

In order to avoid a significant accumulation of errors in the calculation process, this method provides for updating vector functions every 5 steps $\overline{x}(t)$ and $\dot{\overline{x}}(t)$ with high accuracy (about $0(h^6)$). This is done using inequalities:

$$\bar{x}(t_{k+5}) \approx \bar{x}(t_{k+5}) + \frac{5}{24}h^2\cdot\frac{1}{2}(9\cdot\ddot{\bar{x}}(t_{k+4}) + 20\cdot\ddot{\bar{x}}\left(t_{k+1} - 29\cdot\ddot{\bar{x}}(t_k)\right) \tag{37}$$

$$\dot{\bar{x}}(t_{k+5}) \approx \dot{\bar{x}}(t_{k+5}) + \frac{1}{24}h\cdot\left(11\cdot\ddot{\bar{x}}(t_{k+5}) + 5\cdot\ddot{\bar{x}}(t_{k+1}) - 16\cdot\ddot{\bar{x}}(t_k)\right) \tag{38}$$

Thus, in the calculation process, at each step of calculating the values of coordinates and velocities of movement of the moving boundary, the right-hand sides of the system of equations (37) and (38) are calculated only once. This significantly reduces the calculation time compared to other methods, where it is necessary to calculate the right-hand sides of the system of equations several times before taking the next step in time.

As already noted, this circumstance is very large with cumbersome expressions for the right-hand sides of the above system of equations. Given the importance of this circumstance, it is the latter feature that is characteristic of systems of equations describing the dynamics of the hydraulic systems under study.

A comparative analysis of the known methods for numerically solving ordinary second-order differential equations shows that the most widely used method for solving them, the Runge-Kutta method, is a method of only the fourth order of accuracy, it gives an error of the solution of order $0(h^4)$. Here, to take the next step in time, it is necessary to calculate the value of the right-hand side of the system of equations (38) four times. Although the Runge-Kutta method meets the most demanding requirements for the accuracy of numerical methods, its use, however, implies sufficient smoothness of the right-hand sides of equations and their derivatives. It is these conditions that are not met when calculating the characteristics of the processes studied here.

Comparison with other methods of numerical integration of second-order differential equations, for example, with the "polyline" method, shows the disadvantages of this method, since it also requires sufficient smoothness of the obtained solutions. Therefore, it is the Bless method that meets the highest requirements for the accuracy of calculation and the cost of machine time. It also has clear advantages over other methods in terms of universality and applicability to solving systems of second-order equations with discontinuous coefficients.

All these arguments allowed recommending the Bless method for numerically solving problems of dynamics of one-dimensional motion of incompressible liquid media with moving boundaries in the channels of hydraulic lines and machines of the hydraulic systems under study. A deeper and more comprehensive analysis of this method would allow applying it in the future for CAD of promising hydraulic systems as the most accurate, simple, and effective method.

In the introduction, it was noted that when filling channels with narrowing and branches with liquid, phenomena occur in the liquid that significantly affect the dynamics of the filling process of these channels [106, 108]. These phenomena are mainly associated with a narrowing of the fluid flow when filling these channels. When studying these phenomena, difficulties arise in determining the compression ratio ε of the liquid jet in these channels. For local hydraulic resistances of complex geometric shapes with narrowing of channels, this value is necessary to determine the pressure increase at the time of water hammer associated with flow inhibition due to narrowing of the channel when it is filled.

For local hydraulic resistances of complex geometric shapes, difficulties are determined by the fact that the value of the abrupt decrease in the fluid velocity due to impact when filling a channel with this resistance is unknown. The value of the compression ratio of the jet formed at the moment of incomplete hydraulic impact is also not known (Fig. 2). These circumstances do not allow the Zhukovsky formula for incomplete hydraulic shock to be used directly for this case. The simplest scheme of the process under consideration is shown in Figure 2.

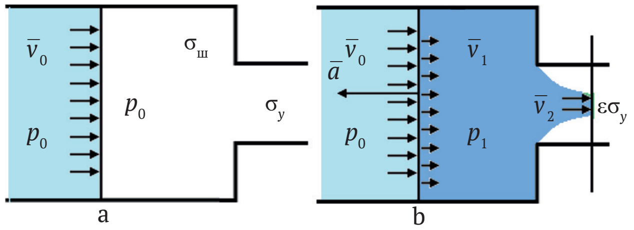

Figure 2. Calculation scheme of incomplete water hammer when filling a channel with a narrowing: a – before the impact, liquid parameters with the number 0; b – after the impact, parameters with the numbers 1 and 2

Figure 2 shows $\bar{a}$ vector of the speed of a water hammer sound wave. The area of fluid after impact is bounded by sections 1 and 2.

Next, the study considers the basic physical picture of the process under study and builds its mathematical model. The necessary relations are obtained for calculating the value ε of the compression ratio of the liquid jet, and the value ΔP of the shock pressure increase when filling a main line with hydraulic resistance in the form of narrowing of the channel with a gas-liquid medium. It is assumed that when filling a compressed liquid or gas-liquid medium of a channel with local hydraulic resistance (in the form of a sharp narrowing of the channel, see Figure 2) the flat front of the liquid flow (before it hits the local hydraulic resistance) moves at a speed of v_0 (Fig. 2a; cross-section 0-0). The pressure of the medium, in this case – air, in front of the liquid flow front is equal to p_0. When the liquid front hits the local hydraulic resistance under study, an increased pressure wave occurs in the liquid p_1 (Fig. 2b; cross-section 1-1). This wave begins to propagate against the current at the speed of sound a. In this case, the speed of movement of liquid particles behind the shock wave decreases by a jump to the value of v_1. Shock pressure increase $\Delta p=p_1-p_0$ in this case, is determined by the Zhukovsky formula for incomplete hydraulic shock:

$$\Delta p=\rho a(v_0-v_1), \tag{39}$$

where ρ – liquid density, kg/m^3.

Speed v_1 is not known. It is known that due to the short duration of the impact, the flow of compressed fluid in the obstacle area with a good approximation can be considered non-viscous and weightless ($Re\to\infty$, $Fr\to\infty$). Therefore, its flow through the narrowing of the channel can be considered in a one-dimensional approximation as a jet with a flat front (Figure 2b, cross-section 1-1, 2-2). The speed v_2 of the flow front is determined by the fluid parameters R_1 and v_1 by the compression wave in Figure 2b this area is more darkened, and also – the value of the coefficient ε of jet compression and pressure R_2 on the free surface of the jet. Therefore, in the region of a compressed liquid, the energy equation can be used in the form of the well-known Bernoulli relation, which here is an expression of the law of conservation and transformation of energy for an ideal liquid:

$$p_1-p_2=\rho(v_2^2-v_1^2)/2. \tag{40}$$

Speed v_1 and v_2 are related here by the ratio of continuity of the liquid flow in the form of equality:

$$v_1\sigma_1=\varepsilon v_2\sigma_2, \tag{41}$$

where σ_1 and σ_2 – the area of the corresponding cross-sections of the channel.

The latter relation is the law of conservation of the mass of the liquid flow for the case under consideration. It should be noted, however, that the front of the fluid filling the trunk is usually frothy and blurry. In addition, with strong geometric narrowing of the channels in local hydraulic resistances at the flow front, even before the impact, a gradual increase in pressure is observed, due to the fact that (as the front approaches the obstacle) the gas in front of the flow front is compressed. These features of the filling process can significantly smooth out impact phenomena. Here, however, for the sake of simplicity of consideration, this is ignored and it is considered that $p_2=p_0$.

To calculate the value ε of the compression ratio of the jet (liquid or gas-liquid flow) at the moment of impact of the flow front on the resistance, and to determine the value Δp in the above cases, the results of experimental studies of ordinary operation of combine harvesters can be used.

Here, the value ε of the compression ratio of the flow jet at the time of incomplete hydraulic impact is proposed to be calculated using the speed values established experimentally from the experiment v_0 at the moment when the front approaches this local hydraulic resistance, and the value Δp of incomplete hydraulic shock. The calculation of the value ε here is proposed to be performed using the equation obtained from the equality, which expresses the well-known laws of conservation of energy and mass, written above. Equation for calculating ε has the form:

$$\varepsilon = \bar{\sigma}\cdot[1+\frac{2\Delta\bar{p}}{M_0(1-\Delta\bar{p})^2}]^{-\frac{1}{2}}, \tag{42}$$

where:

$$\Delta\bar{p}=\frac{\Delta p}{\rho a v_0};\ M_0=\frac{v_0}{a};\ \lambda=(\frac{\bar{\sigma}^2}{\varepsilon^2}-1)^{-1};\ \bar{\sigma}=\sigma_1/\sigma_2\,.$$

Studies have shown that if the value of ε is known in advance (for example, when it is already predefined visually or becomes known from at least one reliable experience, and obtained theoretically or calculated in the above way from experimental data), then it can be used in the future to determine the value Δp of hydraulic shock in a reasonably wide range of changes in full-scale conditions and the above values M_0 and ε.

In this case, the calculation $\Delta\bar{p}$ can use the equation:

$$\Delta\bar{p}=1+\frac{\lambda}{M_0}-[(1+\frac{\lambda}{M_0})^2-1]^{\frac{1}{2}}, \tag{43}$$

where $\lambda=(\frac{\bar{\sigma}^2}{\varepsilon^2}-1)^{-1}$.

Evidently, the above equation is a converted Zhukovsky formula to a form that makes it possible to determine the impact pressure increase Δp in the main line at known values of the flow front velocity v_0 at the moment of impact, the speed of sound a in a liquid, the geometric characteristic of the resistance $\bar{\sigma}=\sigma_1/\sigma_2$ of coefficient ε of jet compression.

Additional studies have shown that the method described above and the calculation equations (42) and (43) can also be used to determine the parameters of incomplete hydraulic shock in the event of a sudden partial overlap of a channel already filled with liquid or gas-liquid medium, for example, by a hydraulic distributor of the working fluid of a volumetric one (Fig. 3).

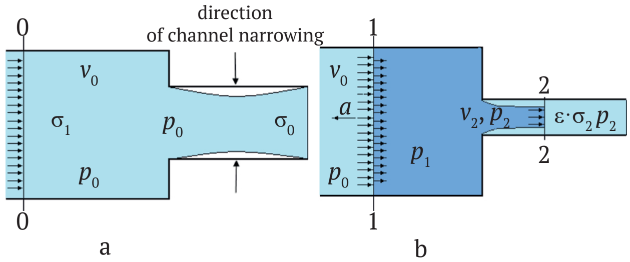

Figure 3. Diagram of fluid movement when the channel is partially blocked:
a – before the overlap (before the impact); b – after the overlap (after the impact)

It is important to note that the coefficient ε, according to the calculation, can be determined using any values of M_0 and $\Delta\bar{p}$ taken from a single experiment for a given local resistance. However, in this case, the question arises about the possible dependence ε from the change of M_0 and $\Delta\bar{p}$.

The studies described in this paper have shown that the values of ε determined experimentally by the equation (42) for washers, practically do not depend on the change of M_0 and $\Delta\bar{p}$. At the same time, the most significant circumstance for practice is that the definition of ε in this way, can be performed on small-sized laboratory models. For example, the value ε obtained for throttle washers with geometric characteristics $\bar{\sigma}$=3.04 and $\bar{\sigma}$=3.31 did not differ by more than 5 % from the above value ε=0.64.

These circumstances allowed determining experimentally (at the second stage of research) the values of ε for complex geometric resistances such as small-sized axial pumps or vane turntables of flow meters. In particular, the value ε for two different types of pumps were determined for small-size laboratory models. They turned out to be equal to 0.50 for the pump with $\bar{\sigma}$=2.53 and 0.56 – for the pump with $\bar{\sigma}$=2.26.

Studies have also shown that the value of the coefficient ε and for small-sized pump models, are practically independent of M_0 and $\Delta\bar{p}$, and can be taken as constant values for the studied values R_e. The values of the jet compression coefficients obtained in this way were used for theoretical calculations of shock pressure increases in various cases of filling with ordinary liquid and liquid containing undissolved gas bubbles.

Using values ε=*const*, dependencies were also calculated for $\Delta\bar{p}=\Delta\bar{p}(M_0)$, which were compared with experimental values $\Delta\bar{p}$. Analysis of these comparisons indicates that the accuracy of calculations of shock pressure increases, in this case, is acceptable for practice (Figure 4).

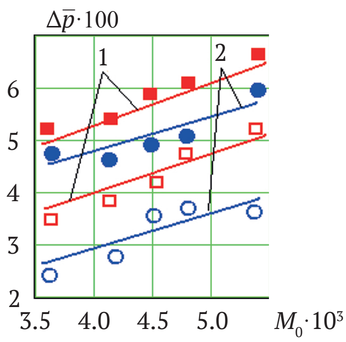

Figure 3. Calculation and experiment data

Note: ■ *– pump,* $\bar{\sigma}$*=2.53;* □ *– pump,* $\bar{\sigma}$*=2.26;* ○ *– washer,* $\bar{\sigma}$*=3.31;* ● *– washer,* $\bar{\sigma}$*=3.04; 1 – calculation for augers; 2 – calculation for washers.*

Thus, as a result of the research described above, elements of the theory were developed and a method for calculating the processes of incomplete water hammer that occurs when filling channels with local hydraulic resistances of complex geometric shapes with liquid or gas-liquid media was created. The developed method allows determining the coefficient ε of liquid jet compression and value Δp^{-} of relative impact pressure increase in case of incomplete hydraulic impact on local hydraulic resistance of complex geometric shape.

CONCLUSIONS

1. Coefficient ε can be defined using any values of M_0 and $\Delta\bar{p}$ taken from a single experiment for a given local resistance. However, in this case, the question arises about the possible dependence ε from the change of M_0 and $\Delta\bar{p}$.

2. The most significant circumstance for practice is that the definition of ε in this way, can be performed on small-sized laboratory models. For example, the value ε obtained for throttle washers with geometric characteristics $\bar{\sigma}$=3.04 and $\bar{\sigma}$=3.31 did not differ by more than 5% from the above value ε=0.64. These circumstances allowed determining experimentally, at the second stage of research, the values of ε for complex geometric resistances such as small-sized axial pumps or vane turntables of flow meters. In particular, the value ε for two different types of pumps were determined for small-size laboratory models. They turned out to be equal to 0.50 for the pump with $\bar{\sigma}$=2.53 and 0.56 – for the pump with $\bar{\sigma}$=2.26.

3. A method of engineering calculation is proposed. The method allows determining the coefficient ε of liquid jet compression and value $\Delta\bar{p}$ of relative impact pressure increase in case of incomplete hydraulic impact on local hydraulic resistance of complex geometric shape hydrostatic transmissions of combine harvesters.

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Аналітичність нестаціонарних процесів зміни діагностичних параметрів гідростатичних трансмісій зернозбиральних комбайнів

Анотація. Створення конкурентоспроможних на світовому ринку сільськогосподарських машин є актуальним завданням. І вирішення цього завдання насамперед пов'язане із питаннями підвищення надійності. Особливо це поширюється на складну, а отже, і дорогу сільськогосподарську техніку. Стає недозволеною розкішшю експлуатувати комбайни, у яких 50 % всього робочого часу припадає на простої. Ці простої пов'язані з недостатнім технічним рівнем та низькою надійністю. Особливо слід зазначити низьку надійність елементів приводів. Одним із напрямів підвищення енергонасиченості складних сільськогосподарських машин є заміна механічних передач для приводу робочих органів на гідравлічні. Збільшення первісної вартості машини за рахунок цього може бути компенсовано зменшенням витрати запасних частин у подальшій експлуатації та скороченням часу простоїв. У конструкціях українських та зарубіжних виробників сільськогосподарської техніки перспективним напрямом на сьогоднішній день є створення багатофункціонального гідромеханічного приводу. Мета дослідження – підтвердити існування аналітичних підходів опису нестаціонарних процесів зміни діагностичних параметрів гідростатичних трансмісій зернозбиральних комбайнів. Методологія основана на принципі Лагранжа, розвинених елементів фундаментальних положень механіки суцільних середовищ з рухомими межами стосовно гідравлічних систем приводів гідростатичних трансмісій зернозбиральних комбайнів, що дозволяють розширити область досліджень і моделювання діагностування зазначених систем. В статті доведено, що в досліджуваній області більш ефективним методом чисельного рішення таких систем рівнянь є метод Блесса. Показано, що для одновимірних рухів нестискуваних рідких середовищ, що переміщаються в каналі і обмежених рухомими межами, розрахунок зводиться до вирішення рівняння $a(x, t)\,\ddot{x}=b(x, t)\,\dot{x}^2+c(x, t)$. Тут $x=x(t)$ – координата переднього або заднього кордону рідкого середовища, що переміщається в каналі. Авторами обґрунтовано, що це рівняння є узагальненим рівнянням Бернуллі на випадок руху нестискуваних рідких середовищ з рухомими межами. Запропонований метод дозволяє здійснити розрахунки динаміки запуску ампулізованої гідросистеми приводу гідростатичних трансмісій зернозбиральних комбайнів з мінімальними обсягами 1...10 % газових порожнин для зберігання робочої рідини приводу. Описані вище елементи теорії і створені методи розрахунку дозволяють розширити область дослідження динамічних режимів роботи гідросистем силового приводу гідростатичних трансмісій зернозбиральних комбайнів в процесі заповнення робочою рідиною каналів гідросистем з відгалуженнями і гідравлічними опорами

Ключові слова: методологія, трансмісія, гідростатика, ефективність, гідравлічні системи