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Forming helical blades from flat blanks with minimal deformation

Abstract. Helical blades are increasingly used in the designs of forage harvesting equipment. They demonstrate better cutting characteristics compared to traditional flat ones, but their manufacture is complicated by the non-developable surface of the surface. The problem of creating an accurate flat workpiece for such knives necessitates the need for a mathematical description of their geometry. The purpose of this study was to determine an analytical method for constructing a flat workpiece for a helical knife, considering the minimum resistance during plastic deformation of the workpiece. To achieve this goal, differential geometry methods were used, in particular, vector analysis of helical surfaces, construction of a Frenet trihedron, and analysis of the first quadratic form of the surface. It was established that the working surface of the knife is a straight open helicoid, which can be bent into a surface of revolution without changing the first quadratic form. Parametric equations of bending of the knife surface using a variable parameter describing the process of transforming the helicoid into a one-sheeted hyperboloid of revolution were constructed. It was proven that the latter is approximated with high accuracy by a truncated cone, the sweep of which is determined by the design parameters of the

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knife. Formulas for calculating the geometric dimensions of the sweep from the known parameters of the knife, in particular the radii of the bases and the height of the truncated cone, were obtained. It was shown that the length of the blade arc and the central angle that outlines the workpiece enable precise determination of its shape. The practical value of the study consists in the creation of an effective method for constructing the most accurate flat workpiece for the manufacture of a helical knife, which allows minimising the resistance during forming, reducing the labor intensity, and increasing the accuracy of manufacturing parts of grinding drums

Keywords: Frenet formulas; curvature; torsion; vector equation of a surface; first quadratic form

INTRODUCTION

Helical blades are widely used in various industries due to their unique shape and ability to process materials efficiently. In particular, they are used in the food industry for grinding and mixing raw materials, in agriculture for processing feed, and in the woodworking and processing industries for grinding wood and secondary raw materials. The main purpose of using such blades is to ensure uniform cutting, improve equipment productivity, and reduce energy costs due to the optimised blade shape.

Helical surfaces find their application in a wide range of technical fields. V. Bulgakov *et al.* (2024) theoretically investigated the traction resistance of harrows with screw working bodies. The authors analysed the factors affecting the efficiency of such harrows, in particular their resistance when performing soil cultivation, which is important for improving agrotechnical characteristics and reducing energy costs in agriculture. The research of S. Liu *et al.* (2023) focuses on optimising the surface texture of screw pairs in screw hydraulic rotary actuator using surrogate methods. The authors used an approach based on surrogate modelling to improve surface parameters, which is important for increasing the efficiency and durability of such systems. M. Mushtruk *et al.* (2020) considered mathematical modelling of the oil extrusion process with pre-grinding of raw materials in a twin-screw extruder. The authors analysed various aspects of the process, in particular in food processing, to optimise the technological process to increase oil yield and reduce energy costs during raw material processing.

Scientists J.C.P. Ortiz *et al.* (2024) focused on optimising the Gorlov screw turbine for hydrokinetic applications using surface response methodology and experimental tests. This contributes to improving the turbine's performance when used in hydroelectric systems, optimising its characteristics for maximum performance and minimum energy consumption. J. Zhao *et al.* (2024) investigated the change in the morphology of tooth surfaces and tribological properties of helical gears during lubricated sliding wear. This made it possible to improve the operational characteristics of helical gears by studying their behaviour under different lubrication and wear conditions, which is important for increasing their durability. The article of H. Liu (2024) was devoted to the theory of formation and computer modelling of rotary cutting tools with helical teeth. The research is aimed at improving the production technologies of such tools, increasing their accuracy and efficiency in material processing processes. J. Chen *et al.* (2024) considered a general coupling model and dynamic analysis of a helical gear pair with tooth surface deviations. This made it possible to evaluate the effect of tooth deformations on the operation of helical gears and their dynamics to improve the accuracy and reliability of such systems in mechanical devices. A. Rucins *et al.* (2024) investigated the energy parameters of a screw conveyor with a bladed working body for transporting agricultural materials to improve efficiency and reduce energy costs when using such systems for moving grain and other agricultural cargo.

As a result of reviewing the sources on the selected topic, it was found that the issue of finding an approximate knife sweep in the form of a flat workpiece, which would be the most accurate and would present the least resistance when forming it into the surface of a helical knife, is relevant. In this regard, the problem of mathematical description of the bending of the helical surface of the knife arises (Filimonov & Bacherikov, 2022). This will make it possible to find a flat workpiece for its manufacture based on a scientific approach.

The article aimed to build an analytical model of the bending of the helical surface of the knife of the grinding drum, with the subsequent determination of an accurate flat workpiece that provides minimal resistance to plastic deformation during forming.

The scientific novelty of the research lies in the use of methods of differential geometry, namely the theory of bending of surfaces based on their internal geometry. Since the surface of a straight open helicoid is non-developable, the study of the bending process can contribute to the improvement of its manufacturing technology.

MATERIALS AND METHODS

The study used symbolic mathematics of the software product "Mathematica", as well as the MatLab software environment for visualisation of the obtained results. To achieve the goal, approaches based on the methodology of differential geometry were used.

One family of coordinate lines of the helical surface was adopted, as helical lines were taken to be helical lines. The helical surface was bent by gradually decreasing its pitch. In this case, the length of the helical lines did not change, as a result of which the radius of the cylinder on which they were located increased. When the pitch of the helical lines became zero, the helical surface was transformed into a surface of revolution. The spatial curve was characterised by two differential parameters – the curvature k and the torsion σ. For a helical line, these parameters were constant and were determined from the following expressions:

$$k = \frac{a}{a^2+b^2};\ \sigma = \frac{b}{a^2+b^2}, \tag{1}$$

where a is the radius of the cylinder on which the helix was located; b is the helix parameter, through which the pitch H was determined $(H = 2\pi b)$. The helix had a pitch angle β, the tangent of which was the coefficient of proportionality between twist and curvature, and which was determined through a and b: $\text{tg}\beta = b/a$. Based on this, another expression was written to determine curvature and twist:

$$k = \frac{\cos^2\beta}{a};\ \sigma = \frac{\sin\beta\cos\beta}{a}. \tag{2}$$

Having solved (1) for the constants a and b, we obtained expressions for finding them from the known curvature k and the torsion σ:

$$a = \frac{k}{k^2+\sigma^2};\ b = \frac{\sigma}{k^2+\sigma^2}. \tag{3}$$

The parametric equations of a helix as a function of the length s of its arc had the form:

$$x = a\cos\frac{s}{\sqrt{a^2+b^2}};\ y = a\sin\frac{s}{\sqrt{a^2+b^2}};\ z = \frac{bs}{\sqrt{a^2+b^2}}. \tag{4}$$

The helical line (4) was taken as the guiding curve through which the helical surface passes. For bending the surface, its vector equation in the accompanying Frenet trihedron of the curve (4) was used. In addition, the first quadratic form of the surface was used, with the help of which the length of the line on the surface was determined. Since the lengths of the lines did not change during bending, the first quadratic form of the surface remained unchanged during its bending. The achievement of such a result indicated that the analytical description of the process of bending the surface was correct.

It is necessary to analyse the shape of the helical knife of the chopping drum of a forage harvester. Figure 1a shows the chopping drum of the New Holland with knives fixed on it, and in Figure 1b – a separate knife with structural dimensions and cross-section. The blade of the knife is a helical line located on a cylinder of radius R with a lift angle $\beta_R = 90° - \tau$. The cross-section of the knife, the thickness of which is conventionally taken as zero, is a segment perpendicular to the axis of the helical line, and which makes an angle φ to the radial direction (Fig. 1c). The surface is formed by the helical movement of this segment at a constant value of the angle φ, therefore the formed surface is a straight open helicoid.

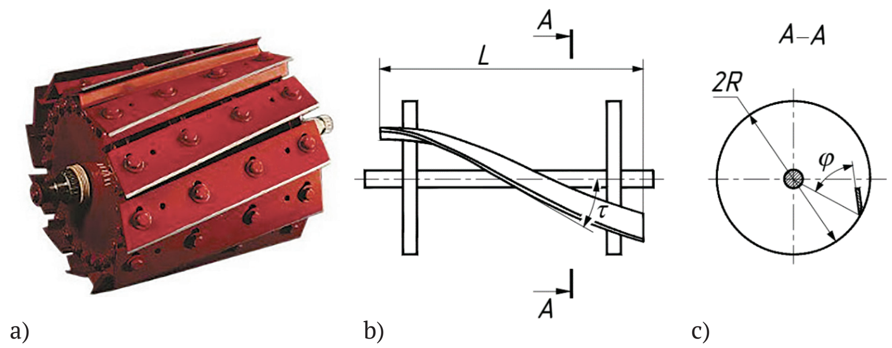

Figure 1. Grinding drum
and design parameters of a separate knife

Note: a) shredding drum with knives; b) separate knife on the drum; c) knife cross-section: L – knife length; τ – angle between the drum axis and the knife blade; $2R$ – drum diameter; φ – angle between the knife cross-section line and the radial direction

Source: a) Pull-type forage harvesters and flail harvester (n.d.); b), c) S. Pylypaka *et al.* (2017)

The guiding curve for forming the surface is taken to be a helix located on a cylinder of radius r and with an elevation angle β. At point A, a Frenet trihedron can be constructed, in which the orthogonal plane $\bar{\tau}$ is tangent to the helix, the orthogonal plane $\bar{n}$ is the main normal of the helix, and the orthogonal plane $\bar{b}$ is the binormal.

RESULTS

Figure 2a shows a frontal projection of a helix with a vertical axis, so the orthogonal plane of the principal normal $\bar{n}$ is projected to a point. The straight-line generators of a straight open helicoid intersect the helix, are parallel to the horizontal plane, therefore, are perpendicular to the axis of the helix, but are transverse to it. This condition is met by the straight-line generator u, which is located in the direct plane of the trihedron parallel to the horizontal plane (Fig. 2a). The direction vector $\bar{w}$ of the straight-line generator in the trihedron system has the coordinates:

$$\bar{w}: \{\cos\beta;\ 0;\ -\sin\beta\}. \tag{5}$$

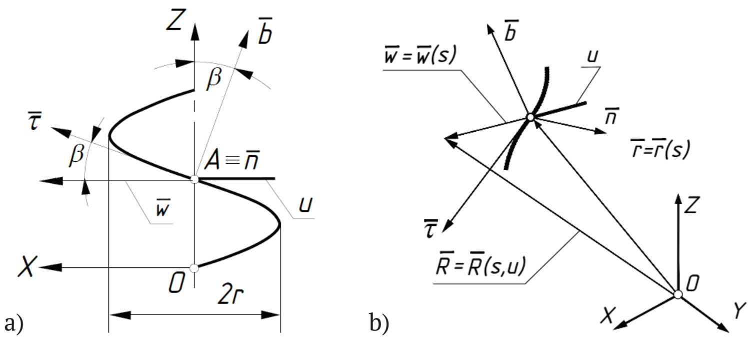

Figure 2. Graphical illustrations for constructing the vector equation of the surface of a straight open helicoid in the Frenet trihedron system of the helical line

Note: a) position of the rectilinear generator u of the surface in the direct plane of the trihedron; b) fragment of the helix and the Frenet trihedron on it with the rectilinear generator: *OX*, *OY*, *OZ* – coordinate axes, units of measurement – linear units; $\bar{\tau}$, $\bar{n}$, $\bar{b}$ – orts of the Frenet trihedron; $\bar{r}=\bar{r}(s)$ – vector equation of the helix $\bar{R}=\bar{R}(s,u)$ – vector equation of a straight open helicoid; $\bar{w}=\bar{w}(s)$ – direction vector of the rectilinear generator, *u* – rectilinear generator; β – angle of elevation of the helix, designated in the Frenet trihedron system concerning its orts; *r* – radius of the cylinder on which the helix is located

Source: developed by the authors based on research conducted

According to Figure 2b, any point on the surface of a straight open helicoid can be expressed as the vector sum of the distance $\bar{r}$ to the vertex of the trihedron and the distance *u* along the direction vector $\bar{w}$:

$$\bar{R}(s,u)=\bar{r}(s)+\bar{\tau}u\,cos\beta-\bar{b}u\,sin\beta. \quad (6)$$

It is known from differential geometry that when a surface is bent, its first quadratic form does not change. It is found through the partial derivatives of the surface (6), taking into account the fact that the angle $\beta=const$:

$$\frac{\partial\bar{R}}{\partial u}=\bar{\tau}\cos\beta-\bar{b}\sin\beta\,;\ \frac{\partial\bar{R}}{\partial s}=\frac{d\bar{r}}{ds}+\frac{d\bar{\tau}}{ds}u\cos\beta-\frac{d\bar{b}}{ds}u\sin\beta. \quad (7)$$

The vector $(d\bar{r})/ds=\bar{\tau}$, the derivatives of the vectors $\bar{\tau}$ and $\bar{b}$ are written in projections onto the vectors of the trihedron according to the Frenet formulas:

$$\bar{\tau}'=\bar{n}k;\ \ \bar{n}'=-\bar{\tau}k+\bar{b}\sigma;\ \ \bar{b}'=-\bar{n}\sigma. \quad (8)$$

After substituting (8) into the partial derivative (7) of the surface along the arc *s* and grouping by vectors can be obtained:

$$\frac{\partial\bar{R}}{\partial s}=\bar{\tau}+\bar{n}u(k\cos\beta+\sigma\sin\beta). \quad (9)$$

The coefficients of the first quadratic form will be written:

$$E=\left(\frac{\partial\bar{R}}{\partial u}\right)^2=cos^2\beta+sin^2\beta=1;\ F=\frac{\partial\bar{R}}{\partial u}\cdot\frac{\partial\bar{R}}{\partial s}=cos\beta\,;$$
$$G=\left(\frac{\partial\bar{R}}{\partial s}\right)^2=1+u^2(k\cos\beta+\sigma\sin\beta)^2. \quad (10)$$

When bending the surface, it is necessary to maintain the condition under which the value of the coefficients (10) should not change. The coefficients *E* and *F* are constant values. The coefficient *G* includes the curvature *k* and the torsion σ. They can be changed so that the expression in parentheses of the coefficient *G* does not change in general. The angle β remains unchanged, since it is included in the coefficient *F*, which also should not change. After bending the surface, the curvature and torsion of the helical base curve will change their value. They can be denoted by k_b and σ_b, respectively. From the condition of invariance of the expression in parentheses of the coefficient *G*, it can be written:

$$k cos\beta+\sigma sin\beta=k_b cos\beta+\sigma_b sin\beta. \quad (11)$$

In equation (11) we can change the curvature k_b or the twist σ_b. The twist σ_b will be changed by multiplying the initial twist σ by the factor *p*: $\sigma_b=p\cdot\sigma$. Substituting into (11) $\sigma_b=p\cdot\sigma$ and solving for k_b will give the result:

$$k_b=k+(1-p)\sigma\cdot tg\beta. \quad (12)$$

Let the initial helix with the angle of rise β on a cylinder of radius *r* be given. Then its curvature and twist are determined according to expressions (2), in which instead of *a*, it is necessary to take *r*. The obtained values of curvature and twist should be substituted into the expressions $\sigma_b=p\cdot\sigma$ and (12) instead of *k* and σ. After simplifications, we can obtain new values of curvature and twist of the helix:

$$k_b=\frac{1}{r}(1-p)\sin^2\beta\,;\ \sigma_b=\frac{1}{r}p\sin\beta\cos\beta. \quad (13)$$

By giving different values of the coefficient *p*, the curvature and torsion of the helix will change, and the pitch angle β will remain constant. At $p=1$, the initial helix will be obtained. To construct a directional helix according to parametric equations (4), it is necessary to find expressions for the constants *a* and *b*. Substituting the expressions for curvature and torsion (13) into formulas (3) gives the result:

$$a=r\frac{(1-p)\sin^2\beta+\cos^2\beta}{1-2p\sin^2\beta+p^2\sin^2\beta};\ b=r\frac{p\sin\beta\cos\beta}{1-2p\sin^2\beta+p^2\sin^2\beta}. \quad (14)$$

At $p=1$ from (14) we can obtain $a=r$, $b=r\cdot tg\beta$, i.e. the parameters of the initial helix. A straight-line generator passes through the helix (4), the direction of which is given by the unit vector (5) in the accompanying trihedron system. Therefore, it is necessary to pass from the trihedron system to the fixed coordinate system in which the helix is located. This can be done using the known transition formulas. After that, it is necessary to pass a straight-line generator of the surface through the helix (4) in the found direction. The parametric equations of the surface take the form:

$$X = a\cos\frac{s}{\sqrt{a^2+b^2}} - \frac{u(a\cos\beta + b\sin\beta)}{\sqrt{a^2+b^2}}\sin\frac{s}{\sqrt{a^2+b^2}};$$
$$Y = a\sin\frac{s}{\sqrt{a^2+b^2}} + \frac{u(a\cos\beta + b\sin\beta)}{\sqrt{a^2+b^2}}\cos\frac{s}{\sqrt{a^2+b^2}};$$
$$Z = \frac{bs}{\sqrt{a^2+b^2}} + \frac{u(b\cos\beta - a\sin\beta)}{\sqrt{a^2+b^2}}, \qquad (15)$$

where u is the second independent variable – the length of the straight-line generator, the count of which starts from the point on the helix. The coefficients of the first quadratic form of the surface (15) take the form:

$$E = \left(\frac{\partial X}{\partial u}\right)^2 + \left(\frac{\partial Y}{\partial u}\right)^2 + \left(\frac{\partial Z}{\partial u}\right)^2 = 1;$$
$$F = \frac{\partial X}{\partial u}\cdot\frac{\partial X}{\partial s} + \frac{\partial Y}{\partial u}\cdot\frac{\partial Y}{\partial s} + \frac{\partial Z}{\partial u}\cdot\frac{\partial Z}{\partial s} = \cos\beta;$$
$$G = \left(\frac{\partial X}{\partial s}\right)^2 + \left(\frac{\partial Y}{\partial s}\right)^2 + \left(\frac{\partial Z}{\partial s}\right)^2 =$$
$$= 1 + \frac{u^2}{2(a^2+b^2)}\left(1 + \frac{(a^2-b^2)\cos 2\beta + 2ab\sin 2\beta}{(a^2+b^2)}\right). \qquad (16)$$

The coefficients E and F coincide with the same coefficients (10). In equations (15), the constants a and b, which are included in the coefficient G, will change during bending according to (14). It is necessary to substitute the expressions a and b from (14) into the expression for the coefficient G (16):

$$G = 1 + \frac{u^2\cos^2\beta}{r^2}. \qquad (17)$$

Thus, none of the coefficients E, F, G includes the parameter p. This indicates that equation (15), where the constants a and b are given in (14), describe a one-parameter set of bendings of a straight open helicoid. If the curvature of the initial helical line from (3) is substituted into the coefficient G (10) at $a=r$, an expression similar to expression (17) will be obtained. This indicates that equation (15), when substituting expressions a and b from (14) into it, describes the process of bending of a straight open helicoid. The parameter p, which is included in the expressions a and b, that is, in the equations of the surface (15), affects the shape of the surface, but does not affect its first quadratic form.

It is advisable to consider the obtained result on the bending of a surface in which the initial helix has a lift angle $\beta=45°$ and is located on a cylinder $r=5$ linear units. In Figure 3, intermediate positions of the surface (frontal projections) are constructed for certain values of the parameter p. The guide helix is thickened. The length of the straight generator is 10 linear units (separate figures are made at different scales for clarity, therefore, the length of the generators on them is not proportional). When bending a straight open helicoid by reducing the torsion of the guide helix, it turns into an oblique open helicoid (Fig. 2b) and with further bending – into a single-cavity hyperboloid of rotation with a torsion equal to zero (Fig. 2c). The reduction of the pitch of the surface turn occurs by reducing the parameter p, that is, by reducing the twist of the guide helix.

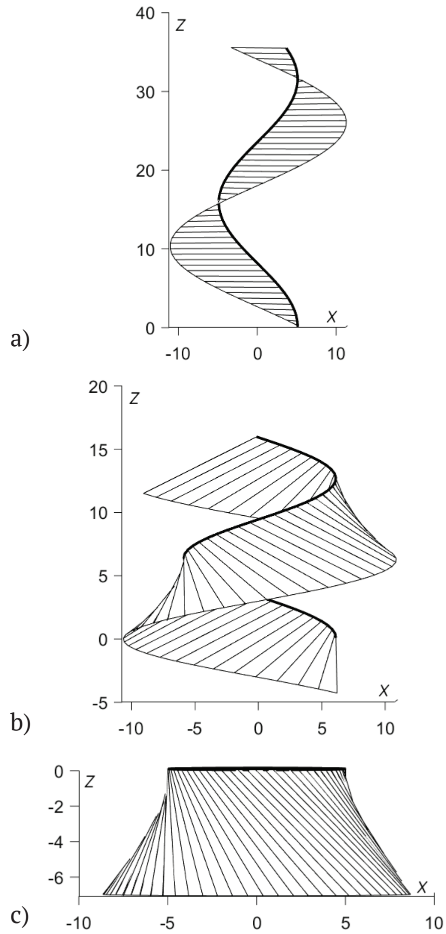

Figure 3. Individual intermediate positions of an open helicoid during its bending

Note: a) $p=1$; b) $p=0.5$; c) $p=0$: X, Z are coordinate axes, units of measurement are linear units

Source: developed by the authors based on research conducted

The task is to find the necessary constants and variable parameters to describe the surface of the knife using equations (15). To do this, we need to find the radius r and the angle β of the helix. The straight-line generator of the surface on the cross-section of the knife is tangent to a circle of radius r (Fig. 4a). The length of the cross-section of the knife u_0 is part of the straight-line generator. From the right triangle, it was found the radius r:

$$r = R\sin\varphi. \qquad (18)$$

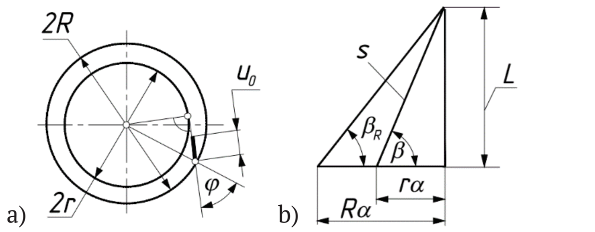

Figure 4. Graphical illustrations for finding the necessary input data for constructing the surface of a helical knife

Note: a) the relationship between the radius of the cylinders on which the helical lines are located; b) the relationship between the angles of elevation of the helical lines: r – radius of the cylinder on which the guiding helix is located; R – radius of the cylinder on which the outer helix of the knife (blade) is located; u_o – width of the knife; φ – angle between the radial direction and the cross-section of the knife; L – length of the knife; s – length of the guiding helix; β – the angle of elevation of the guide helix; β_R – the angle of elevation of the knife blade; r_α – the value of the leg of the triangle on the sweep of the guiding helical line, where α is the angle of rotation of the point during its movement along the guiding helical line, as well as along the blade line within its length; R_α – the value of the leg of the triangle on the sweep of the helical line – the blade of the knife

Source: developed by the authors based on research conducted

From the same triangle, we find the length of the straight line generator to the point on the blade and the limits of change of the parameter u, which correspond to the width of the knife u_o:

$$u = R\cos\varphi - u_0 ... R\cos\varphi. \tag{19}$$

Figure 4b shows helical lines on the sweeps of their cylinders. Through the length of the drum L and the length of the corresponding arc of the circle on the sweep, which are the legs of right-angled triangles, it is possible to determine the tangents of the angles of elevation β and β_R of the helical lines. The angles of rotation α of the ends of the straight generatrix around the axis when moving this generatrix along the helical lines within the distance L are the same. The expressions for the tangents of the angles are written:

$$\text{tg}\beta = \frac{L}{r\alpha};\ \ \text{tg}\beta_R = \frac{L}{R\alpha}. \tag{20}$$

β and β_R can be found taking into account (18):

$$\beta = \text{Arctg}\left(\frac{R}{r}\text{tg}\beta_R\right) = \text{Arctg}\left(\frac{\text{tg}\beta_R}{\sin\varphi}\right). \tag{21}$$

It was needed to find the length s of the arc of the helix on the cylinder of radius r. It can be found from the right triangle through the length L and the angle β, having previously passed from the expression for the tangent β in (21) to the sine β:

$$s = L\text{ctg}\beta_R\sqrt{\sin^2\varphi + \text{tg}^2\beta_R}. \tag{22}$$

When constructing the knife surface, the parameter s changes from zero to the value (22). Let the helical knife be given by the following design parameters: $L = 0.72$ m, $R = 0.25$ m, $\tau = 20°$, $\varphi = 65°$, $u_o = 0.1$ m. The necessary parameters for constructing the knife surface can be found by equations (15). According to (18) $r = 0.2266$ m. By formula (19), the limits of the variable parameter u are determined: $u = 0.006...0.106$ m. The angle of elevation β of the guide helix for the radius r is determined by formula (21), where $\beta_R = 90° - \tau = 70°$: $\beta = 71.75°$. The limits of the change in the length of the helix are determined by formula (22): $s = 0...0.758$ m. These data are sufficient to construct the surface of the knife according to equations (15), taking into account (14) at $p = 1$ and its bending at other values of p. In this case, the radius r acts as a constant a. However, the surface of the one-sheeted hyperboloid of revolution, to which the surface of the knife is bent at $p = 0$, is interesting. The hyperboloid of revolution is constructed in Figure 5, from which it can be seen that it is very close to a truncated cone, which can be considered an approximate sweep of the knife, that is, a flat blank for its manufacture. The cone, in turn, is very close to a cylinder with a height h. From this, we can conclude that the sweep of the knife will be close to a rectilinear strip.

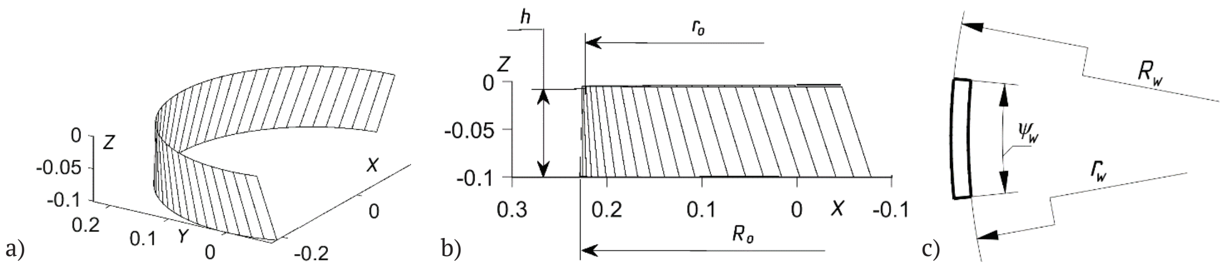

Figure 5. Section of a one-sheeted hyperboloid of revolution on which a knife has bent

Note: a) axonometric image; b) frontal projection with dimensions; c) approximate sweep of the knife: X, Y, Z – coordinate axes, units of measurement – m; R_o – radius of the lower parallel of the one-sheeted hyperboloid of revolution; r_o – radius of the upper parallel of the single-cavity hyperboloid of revolution; h – height of the compartment of the one-sheeted hyperboloid of revolution; R_w – radius of the blade arc on a flat knife blank; r_w – radius of the inner edge arc on a flat knife blank; ψ_w – central angle that limits the flat blank along the length

Source: developed by the authors based on research conducted

It is necessary to find the dimensions of the truncated cone (Fig. 5b). The variable radius ρ, which characterises the helical lines on the surface of the knife and depends on the variable u, must be found at $p=0$. In this case, according to (14) $a=r$, $b=0$. For these data from the first two equations (15):

$$\rho = \sqrt{X^2+Y^2} = \sqrt{r^2+u^2\cos^2\beta}. \quad (23)$$

At $u=0.006$ and $\beta=71.75°$, according to (22) the radius of the upper base of the truncated cone can be found $r_o=0.2266\ m$ (Fig. 5b). Practically $r_o=r$, since $u=0.006\ m$ is a small value. At $u=0.106$ and the same value of $\beta=71.75°$, according to (22), the radius of the lower base of the truncated cone can be found: $R_o=0.229\ m$. The height h is determined from the last equation (15) at $p=0$, (at which $b=0$), as the difference of heights at $u=0.006$ and $u=0.106$. From here it can be found: $h=0.095\ m$. With this data known, as well as the known length s of the upper base of radius r_o ($s=0.758\ m$), it is possible to construct a scan of a truncated cone, which is an approximate flat blank of a helical knife. The described sequence of finding the dimensions of a truncated cone, which approximates the surface of a helical knife, can be generalised. The formulas, which include only the design parameters of the knife, have the form:

$$R_0 = \frac{R\sin\varphi}{\sin\tau\sqrt{\sin^2\varphi+\text{ctg}^2\tau}}; \quad (24)$$

$$r_0 = \sin\varphi\sqrt{R^2+\frac{(u_0-R\cos\varphi)^2}{\sin^2\varphi+\text{ctg}^2\tau}}; \quad (25)$$

$$h = \frac{u_0\text{ctg}\tau}{\sqrt{\sin^2\varphi+\text{ctg}^2\tau}}. \quad (26)$$

Given the design parameters of the knife $L=0.72\ m$, $R=0.25\ m$, $\tau=20°$, $\varphi=65°$, $u_o=0.1\ m$, according to formulas (23), (24), (25), one can obtain: $R_o=0.229\ m$, $r=0.2266\ m$, $h=0.095\ m$. The next stage is finding the sweep of the truncated cone, which will be a flat blank for the helical knife. The dimensions of the sweep are found by the well-known formulas:

$$R_w = \frac{R_0}{R_0-r_0}\sqrt{(R_0-r_0)^2+h^2};$$
$$r_w = \frac{r_0}{R_0-r_0}\sqrt{(R_0-r_0)^2+h^2}. \quad (27)$$

where R_w is the radius of the knife blade on a flat workpiece, r_w is the radius of the inner edge (Fig. 5c). The length of the blade on the workpiece is equal to the length of the larger base of the truncated cone, which is not closed (Fig. 5a). It can be determined from a right triangle (Fig. 4b):

$$s_w = \frac{L}{\sin\beta_R} = \frac{L}{\cos\tau}. \quad (28)$$

From formulas (27), (28), one can obtain: $R_w=9.07\ m$, $r_w=8.97\ m$, $s_w=0.77\ m$. The knife blade on the workpiece rests on the central angle ψ_w (Fig. 5c), which is determined from the expression (for the considered case $\psi_w=0.08$ rad or 4.8°):

$$\psi_w = \frac{L}{R_0\cos\tau}. \quad (29)$$

Given that helical surfaces are widespread in technology, great attention is paid to their manufacture. From an engineering point of view, they all fall under the general definition associated with a helical conoid – the common name in technology of "screw". However, from a geometric point of view, there is a difference between the methods of forming helical surfaces. This difference is taken into account in the study. And it is as follows: in a helical conoid, which is also called a direct closed helicoid, the rectilinear generators intersect the axis of the surface, and in a direct open helicoid, they are parallel to the axis.

It is noteworthy that in the article, the finding of the approximate sweep of the knife surface is closely related to the theoretical description of the process of its bending. According to the theory of differential geometry of surfaces, any helical surface can be bent into a surface of revolution and vice versa. This implies the invariance of the lengths of lines on the surface and its area during the bending process. The surface thickness is assumed to be zero. This is a disadvantage of the method. However, with a relatively small surface thickness, this method gives acceptable results for practice.

In the theoretical description of the bending of helical surfaces in the surface of revolution, it is important to obtain reliable results. In the conducted study, this is confirmed by the coefficients of the first quadratic forms of the surface of the open helical conoid and the one-sheeted hyperboloid of revolution. The correspondence of these forms is accompanied by the found coefficients (10) for the helical surface and the corresponding coefficients (16) for its bending.

The construction of the sweep of a helical knife has its specifics. The fact is that the angle of elevation of the helix (knife blade) is quite large: $\beta_R=70°$. With a drum radius of $R=0.25\ m$, this corresponds to a surface pitch of $H=4.3\ m$. If there were only one knife on the drum, then for its continuous operation, it would have to have a length of $L=4.3\ m$. $L=0.72\ m$ was taken, which requires mounting 6 knives on the drum for continuous operation. The approximate sweep of the sixth part of the full turn of the helical knife is close to a straight strip. The small curvature of the knife blade is also indicated by the radius found on a flat workpiece – $R_w=9.07\ m$. With a blade length of $s_w=0.77\ m$ and such a radius, the workpiece can be replaced with a straight strip. However, the importance of the result is that it is directly related to the process of bending the workpiece into the surface of the knife. As its length increases, the effect of taking into account the curved shape of the flat workpiece will increase.

Numerous studies have been devoted to the construction of developable surfaces. F. Taş & R. Ziatdinov (2023) investigated developable ruled surfaces, which are formed using the curvature axis of a spatial curve. The authors analysed the geometric properties of such surfaces and their construction. The work of A.S.M. Kamarudzaman & M.Y. Misro (2024) is devoted to the comparison of enveloping developable surfaces constructed based on quintic trigonometric Bezier surfaces. The study was conducted in the context of evaluating their geometric properties for use in computer modeling and surface design. T.G. Nelson *et al.* (2019) analysed how developable mechanisms can be integrated into curvilinear structures while maintaining the ability to fold and developable. This opens up new possibilities for creating flexible engineering solutions, particularly in robotics, aerospace, and biomedical devices. The design of a helical blade of an agricultural tool from a planar surface is proposed in the article by S. Pylypaka *et al.* (2024). In this case, the planar surface can be precise, which minimises deformation during bending, but in return, there is a significant limitation of the study. The main difference in the design of surfaces in this work is that the surface is non-planar.

The issue of constructing non-developable surfaces is more complex from a mathematical point of view and is of interest to many researchers. Scientists A. Bankova *et al.* (2024) outlined a method for constructing intersection lines at the transition of sheet material between different surfaces, as well as designing the sweep of such a surface. The research is aimed at improving the processes of designing and manufacturing parts from sheet materials in mechanical engineering and construction. Research of L. Zawallich & R. Pajarola (2024) is devoted to the method of developable 3D models by approximating the mesh using surface flows. The authors propose an algorithm that optimises the developable process of complex geometric shapes, while preserving their structure and minimising deformations.

E. Güler & Y. Yayli (2023) considered the local isometry of generalised helicoidal surfaces in four-dimensional space. The authors investigated the conditions under which such surfaces are locally isometric to each other, which is important for the geometry of multidimensional objects and the theory of surfaces. The article of E. Güler & N.C. Turgay (2024) is devoted to the study of geometric isometries of helical surfaces in five-dimensional Euclidean space. The authors analyse the properties of such surfaces and their behaviour under various transformations, which is important for multidimensional differential geometry and mathematical modelling.

A. Kumar *et al.* (2024) investigated multi-scale surface folding during metalworking. The authors analysed the mechanisms of fold formation and their impact on the surface quality and characteristics of the machined parts, which is important for optimising cutting processes in modern manufacturing. W. Zhang *et al*. (2024) proposed an approach to selecting machining parameters when milling helical surfaces, taking into account spindle vibrations, to increase the efficiency and quality of machining in high-precision machining processes. Research of I. Hevko *et al*. (2018) is devoted to the synthesis of methods for winding screw spirals, which is an important part of the development and improvement of screw mechanism manufacturing technology. The authors analysed various winding methods that allow achieving optimal parameters to ensure the effective operation of screw systems in various industries, such as material processing or agriculture. M. Pylypets *et al*. (2021) analysed the prerequisites for the development of combined operations for the manufacture of screw and auger blanks by the method of metal processing by pressure. The authors considered the latest technologies that combine various processing methods to improve the efficiency of manufacturing such blanks, in particular to reduce material consumption and increase manufacturing accuracy in modern mechanical engineering.

Research of O. Liashuk *et al*. (2019) is devoted to the feasibility study of the manufacturing process of screw working bodies to improve their manufacturing process. S. Pylypaka *et al*. (2017) considered the bounding surfaces of a one-parameter set of planes, as well as methods for their construction and the construction of sweeps. The authors focused their attention on the mathematical and geometric aspects of creating such surfaces, which can be useful for the development of new technologies in mechanical engineering and architectural design. Zh. Li & J. Liqiang (2013) proposed an innovative approach to the design and manufacture of combined propeller blades for use in various industrial applications, in particular in turbine and pump systems. This allowed for to improvement of the technology of manufacturing propeller blades, increasing the accuracy and efficiency of their operation. Ch.-M. Tan & G.-Y. Lin (2016) proposed an innovative mould design for the production of propeller blades for lawn mowers. The authors developed effective methods for optimising the casting process of such blades, which include improving the quality of their manufacture and increasing the productivity of the process.

V. Tarelnyk *et al.* (2019a) investigated the influence of the main technological parameters on the microgeometry, structure, and properties of electroerosion coatings created using combined coatings and surface plastic deformation. The main attention is paid to improving the quality of surface layers by optimising processing modes. Research by V. Tarelnyk *et al.* (2019b) is devoted to the analysis of the stress-strain state of the surface layer after surface plastic deformation of electroerosion coatings. However, the influence of surface geometry on these characteristics was not taken into account at the design stage, which creates prospects for further scientific research in this direction.

Similar to this work V. Khropost & T.A. Kresan (2023) considered finding the sweep of a section of a straight open helicoid due to its bending onto a surface of revolution. However, the description of the bending process was carried out in a Cartesian coordinate system without involving the vector equation of the surface and Frenet formulas. Ultimately, a result similar to that obtained in this study was obtained, i.e., the surface of revolution is a single-cavity hyperboloid. In addition, the construction of an approximate sweep was carried out for an entire turn without giving generalised formulas.

Thus, many scientific works are devoted to the study of the properties, features of geometry, and methods of constructing helical non-developable and expandable surfaces. At the same time, a significant part of these studies is mainly theoretical and is aimed at solving purely mathematical problems, without detailed consideration of the practical aspects of their use in real structures and technological processes. In contrast, within the framework of this study, attention is focused not only on theoretical principles but also on applied aspects, which makes it more oriented towards solving engineering problems.

CONCLUSIONS

A mathematical model of the process of gradual bending of the surface of a helical knife into a surface of revolution has been developed. For this purpose, the theory of differential geometry regarding the bending of helical surfaces is used. It is based on the well-known position that any helical surface can be bent into a surface of revolution by reducing its pitch to zero. In this case, the helical lines are transformed into parallels of the surface of revolution. Based on this position, parametric equations of gradual bending of a straight open helicoid into a surface of revolution have been derived. These include the bending parameter p, which can be given constant values in the range from unity to zero. At $p=1$, the equations describe the initial helical surface, and at $p=0$, the surface of revolution. At intermediate values of the parameter p from unity to zero, the helical surface gradually bends into a surface of revolution. In the obtained equations of gradual bending of the surface corresponding to a helical knife, a transition was made from constant values to the design parameters of the knife. This allowed us to describe the surface of revolution at $p=0$, which is a one-sheeted hyperboloid of revolution, through the parameters of the knife. For example, with a drum radius $R=0.25$ m, an angular elevation of the helix (knife blade) $\beta_R=70°$ and a knife length $L=0.72$ m, a section of a one-sheeted hyperboloid of revolution was obtained with a larger parallel of radius $R_0=0.229$ m and a smaller parallel $r_0=0.2266$ m and a distance between them $h=0.095$ m. These dimensions were taken as the dimensions of a truncated cone, which closely approximates the surface of a single-cavity hyperboloid. After that, an exact sweep of the truncated cone was obtained, which is an approximate sweep of a helical knife. It is a flat ring with an outer radius $R_w=9.07$ m and an inner radius $r_w=8.97$ m.

The obtained result of constructing an approximate sweep of a helical knife is directly related to the mathematical model of the surface bending process, therefore, it is logical. The prospects for further research are to study the surface of helical blades for their improvement from a geometric point of view and simplification of the manufacturing technology.

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Формування гвинтоподібних ножів із плоских заготовок за умови мінімальних деформацій

Анотація. У конструкціях кормозбиральної техніки все ширше використовуються гвинтоподібні ножі, які демонструють кращі характеристики різання порівняно з традиційними плоскими, проте їх виготовлення ускладнюється нерозгортною формою поверхні. Проблема створення точної плоскої заготовки для таких ножів зумовлює потребу в математичному описі їх геометрії. Метою дослідження було визначення аналітичного способу побудови плоскої заготовки для гвинтоподібного ножа з урахуванням мінімального опору при пластичній деформації заготовки. Для досягнення поставленої мети застосовано методи диференціальної геометрії, зокрема векторний аналіз гвинтових поверхонь, побудова тригранника Френе та аналіз першої квадратичної форми поверхні. Встановлено, що робоча поверхня ножа є прямим відкритим гелікоїдом, який може бути зігнутий у поверхню обертання без зміни першої квадратичної форми. Побудовано параметричні рівняння згинання поверхні ножа з використанням змінного параметра, що описує процес перетворення гелікоїда в однопорожнинний гіперболоїд обертання. Доведено, що останній із високою точністю апроксимується зрізаним конусом, розгортка якого визначається через конструктивні параметри ножа. Отримано формули для обчислення геометричних розмірів розгортки за відомими параметрами ножа, зокрема радіусами основ і висотою зрізаного конуса. Показано, що довжина дуги леза та центральний кут, який окреслює заготовку, дозволяють точно описати її форму. Практична цінність дослідження полягає у створенні ефективної методики побудови найбільш точної плоскої заготовки для виготовлення гвинтоподібного ножа, що дозволяє мінімізувати опір при формуванні, знизити трудомісткість і підвищити точність виготовлення деталей подрібнювальних барабанів

Ключові слова: формули Френе; кривина; скрут; векторне рівняння поверхні; перша квадратична форма