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Software and algorithmic support as part of regional systems for monitoring the state of objects for calculation of filtration through earthen hydraulic structures

Abstract. The basis for ensuring the safe operation of hydraulic structures is facility monitoring systems. The introduction of software tools for diagnostic modelling of filtration flows into the software of such systems is an objective step caused by recent real-life events. The aim of this study is to improve the accuracy of existing methods for numerical modelling of two-dimensional stationary filtration flows using the mesh method in the software of the facility monitoring system. The methods used in the study included the mesh method, the finite difference method, as well as approximation methods and numerical algorithms. As a result of the study, it was found that the proposed method of organizing the iterative computing process was effective and useful. Its application can significantly reduce the complexity of software development, as it is based on the formalization and standardization of operations, which simplifies the programming process. In addition, the use of conventional loop statements makes the process of writing programmes more understandable and accessible to developers. The additional array used in the algorithm allows easily changing the configuration of the grid boundaries and the order of operations for each node, which makes the method flexible and suitable for various challenges. A special advantage of the algorithm is its logical simplicity, which contributes to successful adaptation in the case of using multiprocessor systems. Thus, the results of the study confirm the high efficiency and potential of the proposed method for use in practical computing tasks. The

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proposed method of iterative computation is of considerable practical importance in the field of software development for numerical modelling, since its use simplifies the programming process and provides flexibility in working with different problem conditions, which makes it an important tool for a wide range of research and practical applications in hydraulic engineering and related fields

Keywords: object state monitoring systems; grid method; Darcy's law; porous medium; finite difference principle; iterative computing algorithm

INTRODUCTION

Ukraine faces a variety of challenges, including man-made accidents, natural disasters, and other socio-economic crises that require ensuring public safety and societal stability in emergency situations. Such situations can lead to significant human losses, economic damage and negative impact on infrastructure. Early warning of such situations and competent management during the aftermath of the consequences require comprehensive analysis, consideration of various factors and accurate forecasting.

The relevance of this issue is underlined by the growing number of emergencies in the context of military operations and global challenges, which require new strategies and tools to effectively manage risks and minimize possible consequences. In particular, hydraulic structures, which include reservoirs, dams and canals, can become potential sources of danger in emergency situations such as accidents or floods. It is therefore important to develop mathematical models to predict risk and manage such situations. Such mathematical models can allow analysing different scenarios, calculating the probability of negative consequences and developing optimal response strategies (Kupin *et al.*, 2018).

It is important to bear in mind that failure to consider such important aspects can lead to serious consequences. Insufficient forecasting and management of emergencies related to hydraulic structures can lead to human casualties, material damage and environmental disaster. Lack of proper planning and effective response measures can exacerbate the situation and increase the threat to human life and health. Thus, research on the development and improvement of mathematical models for predicting emergencies related to hydraulic structures is an urgent and critical task. Understanding and studying these aspects will ensure an effective system of public protection and minimize risks to life and economy in the event of emergencies.

According to research by V. Pryshlyak & V. Dubchak (2020), in order to predict the possibility of emergencies at hydraulic structures, one of the crucial areas is the use of mathematical modelling methods, such as methods for numerical solution of the Navier-Stokes equations. It is noted that such equations are used to model the movement of fluid in various hydraulic systems, including rivers, reservoirs, and canals. These methods allow analysing and predicting the movement of water, pressure distribution at different points of the system, which helps in managing and preventing possible negative consequences, such as the destruction of hydraulic structures.

The study by Y. Ivanchuk *et al*. (2019) also noted that the method of hydrodynamic modelling using the Navier-Stokes equations can be used to simulate hydrodynamic phenomena in rivers and reservoirs. Importantly, this method can be highly effective in reproducing complex hydrodynamic processes, such as turbulent flows, current interactions, and the formation of hydrological regimes. This makes hydrodynamic modelling an important tool for predicting hydrodynamic processes and developing effective water management strategies.

In the study by O. Brazaluk *et al.* (2023), boundary element methods based on the idea of solving the equations of fluid motion at the boundary of a domain were used to model the movement of water in rivers and reservoirs. These methods include approximation of internal values by finding exact or approximate solutions only at the boundary of the domain. It is emphasized that such methods avoid the complexity of solving equations in the entire domain, reducing the amount of computation and improving the efficiency of numerical modelling of hydrodynamic processes.

In the work of O. Zakharchuk (2022), the author notes that the finite difference method can be used to build numerical models of water movement in reservoirs, taking into account various hydrodynamic phenomena. These models can accurately reproduce the movement of fluid under various conditions, including the impact of hydraulic structures and thermodynamic processes. Importantly, the finite difference method can also be used to solve heat transfer equations, which allows analysing the temperature fields of water in hydraulic systems.

According to O. Vorontsov *et al.* (2022), the finite difference method has several significant advantages. As a result of the work, it is noted that one of the main advantages is its high versatility and flexibility, which allows it to be used for modelling. In addition, this method can provide stable and reliable solutions in a wide range of conditions, including complex geometries and different physical properties of the medium. Its use allows for fast and efficient results, especially when the computational parameters are properly selected (Miroshnyk *et al.*, 2023). Since the above-mentioned studies did not pay sufficient attention to the numerical modelling of two-dimensional stationary filtration flows by the grid method, the aim of this work is to improve these known methods. This work is based on an in-depth study of the principles and algorithms of numerical modelling of filtration flows, which allows increasing the accuracy and efficiency of prediction of hydrodynamic phenomena.

MATERIALS AND METHODS

In the course of this study, such methods as the grid method for solving two-dimensional stationary filtration flows, the finite difference method, as well as approximation methods and numerical algorithms for calculating the values of the potential function under physical conditions were used. The numerical modelling of two-dimensional steady-state filtration flows in hydraulic systems was carried out using the grid method. Firstly, the region in which the solution was to be sought was considered and approximated by a grid with a boundary. After that, the grid spacing was selected, which allowed the differential equations to be replaced by difference analogues. A five-point template was used to replace the second-order derivatives.

For each internal grid node, the value of the potential function was calculated as the arithmetic mean of the function values in neighbouring nodes located at a distance of step h. This iterative process was organized in such a way that for each internal and external grid node, an additional grid node was matched. Each node of this grid had a specific value that determined the actions and arithmetic operations for each node of the main grid. This approach reduced the complexity of software development by simplifying the process of writing code through the use of common loop statements and formalization of operations. The use of an additional array made it easy to change the configuration of the grid boundaries and the order of operations for each node. The logical simplicity of the algorithm contributes to its successful adaptation for use in multiprocessor systems.

The method of finite differences was used to numerically approximate the differential equations describing physical processes in hydraulic engineering systems. This method is based on the approximation of derivatives by difference expressions and their subsequent use to calculate the values of functions on a grid of points. By dividing the domain into a grid and replacing the derivatives with difference analogues, the problem of the differential equation was transformed into a system of algebraic equations that could be solved and numerical results obtained. The finite difference method made it possible to study the distribution of potentials, pressures and other parameters in space and time, which is important for the correct understanding and modelling of hydrodynamic phenomena in hydraulic structures.

The least-squares method and the method of successive approximations were also used to optimize the parameters of the mathematical models used in hydraulic engineering studies. They allowed selecting parameter values that best fit the required data, ensuring the accuracy and reliability of the modelling results. The least-squares method was used to build regression models and solve linear approximation problems, while the method of successive approximations was used to iteratively solve nonlinear problems and converge to the optimal solution. The use of these methods helped to improve the accuracy and reliability of the research results, and made the models more adaptable to different conditions and input data.

In addition, various approximation methods and numerical algorithms were also used to achieve the goal (Hulianytskyi & Riasna, 2017). They were used to approximate continuous functions, such as domain boundaries or boundary conditions. This approach made it possible to obtain a discrete representation of the continuous quantities required to solve the equations on the grid.

Finally, the study used various numerical methods for solving Laplace's equations and other differential equations. These methods were used to approximate the derivatives and numerically solve the equations at the internal points of the grid. The use of a combination of different methods allowed obtaining more accurate and efficient results in modelling steady-state filtration flows in hydraulic systems. Each of these methods contributed to the solution of the problem and improved the quality and accuracy of the calculations.

RESULTS

Hydraulic structures in Ukraine are an important element of the engineering and technical infrastructure aimed at solving a number of key tasks related to the management of the country's water resources. They were built to prevent floods, which have historically caused huge material damage to the national economy and resulted in human casualties. Floods that regularly occur in Ukraine are often catastrophic, so the creation of hydraulic structures has become a necessity to ensure the safety of the population and the country's economic development (Nikitenko, 2020).

Another important function of hydraulic structures is to regulate the flow of rivers, such as the Dnipro, Dniester, Southern Bug, Siverskyi Donets and others. River flow requires attention and control, as uncontrolled water levels can lead to flooding, bank erosion and other negative consequences. Hydraulic structures provide the ability to regulate water flows, which helps prevent excessive flooding or drying up of rivers, maintaining ecological balance and ensuring the rational use of water resources. It should also be borne in mind that hydraulic structures perform an important function in providing water to certain regions of the country. Rural and urban settlements, industrial facilities, as well as agricultural land, require stable access to water for drinking water supply, production needs and irrigation. Hydraulic structures, such as reservoirs and canals, provide the ability to regulate the distribution of water resources, which is important for the sustainable development of various sectors of the economy and the livelihoods of the population.

The efficient operation of industrial enterprises is extremely important for the economic development of a country. However, even with full compliance with regulations, a completely safe environment at industrial facilities is not possible. For this reason, the Civil Protection Code of Ukraine (2013), Law of Ukraine No. 2245-III "On High Risk Facilities" (2001) and Order of the Ministry of Regional Development, Construction, Housing and Communal Services of Ukraine No. 29 "On Approval of DBN B.2.5-76:2014

'Automated Systems for Early Detection of Emergency Threats and Public Warning'" (2014) stipulate the need to implement automated systems for early detection of potential threats and rapid notification of the public about possible emergencies. These measures are aimed at minimizing risks and improving safety at industrial facilities, which is an important step in ensuring stability and protecting the public.

Hydraulic structures are a complex system of engineering structures aimed at optimizing the use of water resources and reducing the negative impact of the water element. They include a variety of structures such as dams, weirs, canals, locks, tunnels, and others that are used for a variety of purposes, from power generation to improving navigation and ensuring water supply (Table 1).

Table 1. Emergencies at Ukrainian hydraulic structures

Waterworks	Date of the emergency	Reasons	Consequences
Dnipro HPP	September 1948	Technical fault, deficiencies in operation	Large human losses and serious material damage
Kaniv HPP	1979	Equipment malfunction, technical failure	Human casualties and material damage
Kremenchuk HPP	2009	Technical failure, improper maintenance	Large flooding on the territory of the power plant
Dnipro HPP	2014	Poor coordination, weather conditions	Damage to infrastructure, flooding
Horodok HPP	2017	Poor technical structure, natural factors	Flooding of territories, damage to infrastructure
Dnipro HPP	2019	Insufficient planning work, natural factors	Flooding of the surrounding areas, evacuation of residents
Kaniv HPP	2021	Insufficient planning work, natural factors	Flooding of the surrounding areas, damage to infrastructure

Note: HPP – hydroelectric power plant
Source: V. Sheremet & M. Kravtsov (2019)

Tailings ponds, as one of the hydraulic structures, play an important role in the storage and treatment of various types of waste, including radioactive, toxic and other wastes, as well as mineral processing. However, their importance is accompanied by potential hazards, as accidents can have serious consequences. Hydrodynamic accidents that can occur at hydraulic structures lead to the formation of artificial floods and breakthrough waves. These phenomena can lead to the destruction of buildings and structures, as well as the loss of life and animals, and cause significant environmental damage. If radiation and chemical hazardous facilities are present in the area of the breakthrough wave, chemical and radioactive contamination zones may be formed. This could lead to serious consequences for human health and the environment. In addition, the destruction of hydraulic structures can cause fires and explosions due to short circuits in electrical networks and damage to fire and explosion hazardous facilities. Therefore, hydraulic structures, playing an important role in the use of water resources and combating the harmful effects of water, require a systematic approach to safety and risk management to prevent potential emergencies and minimize their consequences.

Ukraine's hydropower facilities typically use earthen dams made of sand, sandy loam, loam, clay, gravel, pebble, and their mixtures, as well as rubble stone, which play an important role in controlling water levels and managing water flows. Filtration is a process characterized by the slow movement of liquids or other substances through a porous medium under the influence of various factors. In hydraulic structures, earthen dams in particular act as filtration barriers that allow water to pass through them (Maggay *et al.*, 2021).

Taking into account the above factors, it should be noted that the development of new mathematical models of the filtration process and the improvement of mathematical modelling algorithms play a critical role in ensuring the efficiency and safety of hydraulic structures in Ukraine. However, the understanding and forecasting of these processes needs to be constantly improved and enhanced. New mathematical models and algorithms allow better understanding the dynamics of seepage flows, predict their development and effectively intervene in the event of emergencies such as floods. Such new models and algorithms can form the basis for developing more accurate and efficient monitoring and control systems for hydraulic structures. They will help to ensure a prompt response to changes in filtration processes and make informed decisions in a wide variety of situations that may arise during the operation of hydraulic structures. When analysing filtration flows in hydraulic structures, it is important to use effective numerical analysis methods that allow obtaining accurate solutions to complex mathematical problems. One of these methods, the method of the theory of functions of a complex variable, can be applied in some cases, but it has its limitations.

The most versatile methods are numerical methods, including the mesh method, the finite element method, the method of major domains, the method of summation representations, the method of dummy domains, and others. For example, the mesh method is used to approximate differential equations in different parts of space, which allows obtaining a numerical solution to the problem. The finite element method is used to divide a region into a finite number of elements, where a differential equation is solved on each element. At the same time, each of these methods has its advantages and disadvantages that must be taken into account when using them (Long *et al.*, 2021). Thus, the development of new and improvement of existing methods for numerical analysis of filtration flows is an important scientific task. This will improve the accuracy and speed of solving complex problems, as well as ensure more efficient management and decision-making in the field of hydraulic structures.

Mathematical models of filtration flows are usually based on Darcy's law. This law was discovered and empirically established by the French engineer Henri Darcy in the mid-19th century. He established the relationship between the filtration rate and the hydrodynamic pressure gradient in a porous medium. Darcy's law allows describing the process of filtration through a porous medium and understand its characteristics, such as the rate of seepage of a liquid or gas (Suárez-Grau, 2022). This law is fundamental in hydrodynamics and hydrogeology, and it is an important tool for solving various engineering problems related to water flow and filtration, as shown in (1):

$$v = kJ = -k\frac{dH}{ds}, \tag{1}$$

where: k is the filtration coefficient; J is the piezometric pressure gradient along the path S.

In this case, (2, 3):

$$J = -\frac{dH}{ds}, \tag{2}$$

$$H = \frac{P}{\rho g} + z(\rho g = \gamma), \tag{3}$$

where p – hydrostatic pressure acting on the liquid; ρ – density; γ – specific gravity; g – gravitational acceleration; z – height relative to the initial plane.

Thus, according to Darcy's law (1), the head loss during filtration through a porous medium is proportional to the velocity of the liquid or gas through the medium. This means that the higher the filtration rate, the greater the head loss. It is also important to note that the filtration coefficient k is a parameter that determines the rate of seepage of a liquid or gas through a porous medium. It depends on various factors, including the properties of the porous medium, as well as the fluid that is filtered through it (4):

$$k = \frac{k_f \rho g}{\mu}, \tag{4}$$

where k_f – is the filtration permeability of the medium; ρ is the mass density of the liquid; μ is the dynamic viscosity parameter; g is the gravitational acceleration.

The diversity of porous media and the liquids that seep through them results in different filtration coefficients. The filtration coefficient of water, for example, can vary depending on soil properties, depth of liquid penetration, pressure and other factors. For this reason, typical values of this coefficient are often used for ease and accuracy of design and calculation (Table 2).

Table 2. Typical values of the filtration coefficient for different soils

No.	Type of soil	Filtration coefficient	Unit of measurement
1	Gravel	10^{-2}-10^{-1}	cm/s
2	Coarse sand	10^{-2}-10^{-1}	cm/s
3	Fine sand	10^{-3}-10^{-2}	cm/s
4	Sandy loam	10^{-5}-10^{-4}	cm/s
5	Loam	10^{-7}-10^{-6}	cm/s

Source: A. Arifjanov *et al.* (2023)

When it comes to filtration in an anisotropic medium, where the properties of the porous structure can vary depending on the direction, certain peculiarities arise that should be taken into account. For example, the linear relationship between the filtration rate and the pressure gradient remains valid. That is, as the pressure gradient increases, the filtration rate also increases, and vice versa. However, it is worth noting that in the case of an anisotropic medium, the vectors corresponding to the direction of the pressure gradient and the filtration rate may not remain collinear. Thus, the direction in which the liquid or gas moves may differ from the direction in which the pressure in the medium changes. Such differences need to be taken into account when analysing and modelling filtration processes in anisotropic media for more accurate results and predictions. A key role in creating the foundation for mathematical modelling of hydromechanical filtration processes was played by a system of equations to describe various aspects of fluid movement in porous media (Khujaev *et al.*, 2019). This system of equations allows modelling filtration processes with high accuracy and allows taking into account various conditions and properties of the medium (5-9):

$$\frac{1}{g}\frac{\partial v_x}{\partial t} + \frac{\partial H}{\partial x} + \frac{1}{k}v_x = 0, \tag{5}$$

$$\frac{1}{g}\frac{\partial v_y}{\partial t} + \frac{\partial H}{\partial y} + \frac{1}{k}v_y = 0, \tag{6}$$

$$\frac{1}{g}\frac{\partial v_z}{\partial t} + \frac{\partial H}{\partial z} + \frac{1}{k}v_z = 0, \tag{7}$$

$$n\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0, \tag{8}$$

$$f(\rho, H, T) = 0. \tag{9}$$

These models were proposed in the form of a system of closed equations that allows describing non-stationary processes of fluid filtration in undeformed soil. However, it is important to note that these equations are applicable only in the case of Darcy's law. Particular attention should be paid to the continuity equation and the equation of state. Thus, the continuity equation (8) takes into account the soil porosity (n), which is an important characteristic for determining the volume fraction of pores during filtration. In addition, the equation of state (9) establishes the relationship between the fluid density (ρ), isothermal head (H) and absolute temperature (T). This allows for a detailed description of the fluid state and the relationship between the various parameters, which is important for proper modelling and understanding of filtration processes.

Integration of the system of equations (5-9), taking into account specific boundary and initial conditions, within the area where the filtration process takes place, allows obtaining the values of projections of the filtration rate, piezometric head and liquid density. The projection of the filtration rate indicates the rate of movement of the liquid through the pores and channels of the soil. The piezometric head reflects the pressure generated by the difference in liquid levels at different points in the porous medium. Fluid density allows determining the mass of fluid present in a given volume of porous media, which is important for understanding and modelling filtration processes. This approach helps to understand the physical processes that occur in a porous medium during filtration and to establish relationships between various parameters of these processes. In the case of incompressible homogeneous liquids, the equation of state is given by (10):

$$\rho = const. \tag{10}$$

In this case, the continuity equation will look like (11):

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0. \tag{11}$$

In the case where all parameters remain constant, i.e., with stationary filtration, using equations (5-7), (12-14) will be obtained:

$$v_x = -k\,\frac{\partial H}{\partial x}, \tag{12}$$

$$v_y = -k\,\frac{\partial H}{\partial y}, \tag{13}$$

$$v_z = -k\,\frac{\partial H}{\partial z}. \tag{14}$$

If substituting equation (12) into equation (11), the result is (15):

$$-\frac{\partial}{\partial x}\left(k\frac{\partial H}{\partial x}\right) - \frac{\partial}{\partial y}\left(k\frac{\partial H}{\partial y}\right) - \frac{\partial}{\partial z}\left(k\frac{\partial H}{\partial z}\right) = 0. \tag{15}$$

Thus, when considering the filtration of a liquid in a porous medium, the head *H* can be described by a homogeneous elliptic equation. This is possible provided that the filtration coefficient *k* remains constant, indicating that the soil is isotropic and homogeneous. In this case, the equation describing the head distribution takes the form of the Laplace equation (16):

$$\Delta H = \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = 0. \tag{16}$$

Also, if defining the function φ as follows (17):

$$\varphi = -kH. \tag{17}$$

Then, at a constant value of *k* (*k* = *const*), using equations (15) and (16), we have (18):

$$\Delta\varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0. \tag{18}$$

That is, it should be noted that this function φ is harmonic within the filtering region. Given this information, it is important to understand that the points where the function φ has a constant value, form equipotential surfaces. In the context of equation (17), these equipotential surfaces are essentially surfaces of equal head. Thus, each point on such a surface will have the same head. At the same time, taking into account equation (17), it can be understood that the filtration velocity vector at any point within the filtration area will be directed perpendicular to the surface of equal head. This means that the direction of the water flow at these points will be directed along the potential lines corresponding to the gradient vector of the function φ.

When considering the system of equations (5-9) in the case of two-dimensional liquid filtration, the specific conditions of this process in a limited area of space are taken into account. In the context of in-plane filtration, when the process takes place in a plane parallel to the *XOY* coordinate plane, the system of equations takes the following form (19-21):

$$v_x = -\mathrm{k}\,\frac{\partial H}{\partial x}, \tag{19}$$

$$v_y = -\mathrm{k}\,\frac{\partial H}{\partial y}, \tag{20}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0. \tag{21}$$

In this case, equation (18) will take the form (22):

$$\Delta\varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0. \tag{22}$$

Thus, when considering plane filtration flows, it will be useful to use the flow function ψ equal to (23, 24):

$$V_y = \frac{\partial \psi}{\partial y}, \tag{23}$$

$$V_x = \frac{\partial \psi}{\partial x}. \tag{24}$$

In the case of such a description of ψ, the continuity equation (21) actually becomes an identity, and equations (12-14) yield (25, 26):

$$\frac{\partial H}{\partial x} = -\frac{1}{k}\,\frac{\partial \psi}{\partial y}, \tag{25}$$

$$\frac{\partial H}{\partial y} = \frac{1}{k}\,\frac{\partial \psi}{\partial x}. \tag{26}$$

Also, given that *k* = *const* and the function φ (17), it turns out that φ is related to ψ as follows (27, 28):

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}, \tag{27}$$

$$\frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x}. \tag{28}$$

After differentiating from equations (25) and (26), and then subtracting equation (26) from (25), the following expression (29) is obtained:

$$-\frac{\partial}{\partial x}\left(\frac{1}{k}\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{1}{k}\frac{\partial \psi}{\partial y}\right) = 0. \tag{29}$$

So, it turns out that this equation describes the flow function ψ in the filtering region. It is also important that the flow lines, where the value of the flow function is constant ($\psi = const$), are the key to analysing this motion. These lines allow tracking the direction of fluid flow at each point in the filtration area: the filtration velocity vector is always tangential to the flow line that passes through that point. At the boundaries of the filtration area, where the fluid does not penetrate, the normal component of the velocity should be zero, reflecting the impermeability of these surfaces (30):

$$V_n = 0. \tag{30}$$

In practical applications of modelling filtration processes, the mesh method is often used (Zhu *et al.*, 2021). This method is known for its effectiveness in transforming systems of partial differential equations into systems of linear algebraic equations, which greatly simplifies their solution. In most cases, the solution using this method is carried out in 3 stages:

1. The first step in using the mesh method is to replace the domain of continuously varying arguments with a finite, discrete set of points. For the two-dimensional case, this set forms a difference grid containing both internal nodes and nodes on the boundaries. The equations are usually solved for the internal nodes, and the values of the functions at the boundary nodes are set according to the boundary conditions of the problem.

2. Next, the differential equations are replaced by their difference analogues, which leads to a system of algebraic equations. The number of these equations is proportional to the number of internal nodes of the difference grid.

3. The resulting system of algebraic equations is usually of large order. The solution of such a system can be achieved by various methods, but in practice, an iterative approach is often used to find a solution.

To illustrate the method of meshes for solving systems of equations, we should consider the Dirichlet problem for a classical elliptic equation, namely the Laplace equation. This problem involves setting boundary conditions to determine the solution within a given domain:

$$\varphi(x, y)|_G = \varphi_0(x, y), \tag{31}$$

where *iG* – is the boundary of the domain in which the solution is to be found, and (x, y) satisfies equation (22) and boundary condition (31) (Fig. 1). To compute the solution on this domain, it is approximated using a mesh. The domain *G* is replaced by an approximate domain, G_h which consists of the internal nodes of the mesh and the boundary G'_h.

At the first stage of mesh formation, the domain *G* of the continuously varying argument is replaced by a mesh G_h where each internal node corresponds to a specific solution value. Given the boundary conditions, the boundary G'_h which corresponds to the boundary of *G*. The grid spacing can be different in different directions, but most often it is the same in the *x* and *y* axes. This simplifies the calculations and makes them more efficient. Also, keep in mind that if the grid spacing is too large, it may lead to insufficient accuracy of the solution.

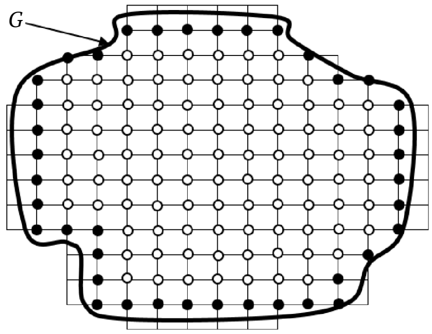

Figure 1. The domain G and its boundary, in which it is planned to search for a solution to the system of equations
Source: developed by the authors

In the context of the mesh method, nodes are the intersection points of lines with fixed *x* and *y* values. These nodes divide the area *G* into parts, which simplifies the calculation of values at these points. In general, there are internal and external (or boundary) nodes. The inner nodes are located inside the domain *G* and have four neighbouring nodes that also belong to this domain. In Figure 1, they are denoted as circles. The outer nodes, in turn, lie on the edge or boundary of the region *G*. They have only a limited number of neighbours, since one or more sides of these nodes may be outside the region *G*. On the grid, these nodes are marked with black circles to distinguish them from the interior nodes.

When proceeding to the numerical solution of the Laplace equation, the Laplace operator is usually replaced by a difference operator for computing on a grid. To replace the second-order derivatives included in the Laplace operator, we used a five-point calculation template based on the function values at neighbouring grid nodes (Fig. 2). This template allows approximating the second derivatives with sufficient accuracy. In this case, it is usually assumed that the grid spacing along the *x* and *y* axes is the same and equal to *h*. This simplifies the calculations and ensures uniformity in the numerical method, which makes it easier to solve the equation on a computer (Catino *et al.*, 2023; Nguyen & Phuc, 2023).

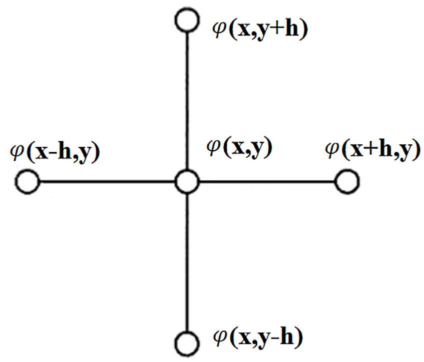

Figure 2. Five-point calculation pattern based on five points
Source: developed by the authors

Expanding the function into a Taylor series around (x, y) yields (32, 33):

$$\varphi(x \pm h, y) = \varphi(x, y) \pm h\frac{\partial\varphi}{\partial x} + \frac{h^2}{2!}\frac{\partial^2\varphi}{\partial x^2} + \\ + \frac{h^3}{3!}\frac{\partial^3\varphi}{\partial x^3} + O(h^4), \tag{32}$$

$$\varphi(x, y \pm h) = \varphi(x, y) \pm h\frac{\partial\varphi}{\partial y} + \\ + \frac{h^2}{2!}\frac{\partial^2\varphi}{\partial y^2} + \frac{h^3}{3!}\frac{\partial^3\varphi}{\partial y^3} + O(h^4). \tag{33}$$

Then, from equations (32) and (33), (34, 35) are obtained:

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\varphi(x+h,y) - 2\varphi(x,y) + \varphi(x-h,y)}{h^2} + O(h^2), \tag{34}$$

$$\frac{\partial^2\varphi}{\partial y^2} = \frac{\varphi(x,y+h) - 2\varphi(x,y) + \varphi(x,y-h)}{h^2} + O(h^2). \tag{35}$$

Also, when using a five-point pattern, the Laplace operator will look like (36):

$$\varphi(x+h, y) + \varphi(x+h, y) + \varphi(x+h, y) + \varphi(x+h, y) - 4\varphi(x, y) = 0. \tag{36}$$

Then, equation (36) is written in the form (37):

$$\varphi(x, y) = \frac{1}{4}(\varphi(x+h, y) + \varphi(x+h, y) + \varphi(x+h, y) + \varphi(x+h, y)). \tag{37}$$

When implementing the iterative process to solve the equation on the grid, for each internal node, the value of the potential function $\varphi(x, y)$ is calculated as the arithmetic mean of the values φ in the nodes located at a distance h from the central node. The proposed algorithm of iterative calculations for determining the values of the potential function for internal nodes at given values for external (boundary) nodes involves the use of an additional grid that has the same shape and dimension as the main difference domain G. Each node of this additional grid has a certain value, which is set before the start of the calculation and is used as a control sign. For each inner and outer node of the main grid G, it is proposed to establish a correspondence with a node of the additional grid. This auxiliary grid node determines what actions and arithmetic operations should be performed for each node of the main grid G. Thus, the use of an auxiliary grid allows effectively managing the computation process and provides flexibility in choosing operations for each node of the main grid depending on its location and task properties.

The proposed algorithm can be implemented in different programming languages. The algorithm shown in Figure 3 implements a fragment of the Python program from Figure 4. To verify the performance of the proposed algorithm, the depression line of an earthen dam on an impermeable foundation was calculated.

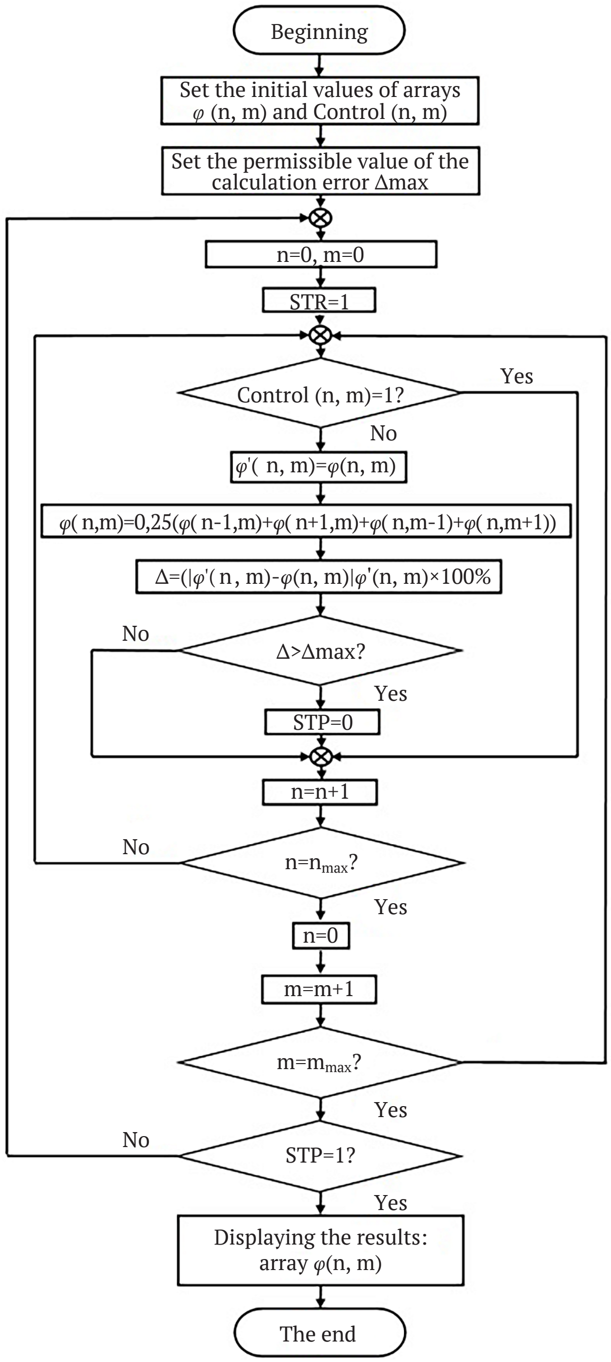

Figure 3. Algorithm of the iterative computing process
Source: developed by the authors

````
```
python
errors = {"cycle":[0],"value":[100]}
k = 0
error = 100
accuracy = 0.005
dataMatrix = input
while error > accuracy:
    error = 0
    k = k + 1
    newDataMatrix = np.zeros((rows,columns))
    for i in range(rows):
        for j in range(columns):
            if control[i,j] == 0:
                # print("i:",i,"j:",j)
                if(i==1 and j == 16):
                    # print("Cycle:",k,"OLD",dataMatrix[i,j],"NEW",newDataMatrix[i,j])
                newDataMatrix[i,j]              =              (dataMatrix[i+1,j]+dataMatrix[i-
1,j]+dataMatrix[i,j+1]+dataMatrix[i,j-1])/4
                if abs(newDataMatrix[i,j]) == 0:
                    error = 100
                else:
                    tempError = (abs(newDataMatrix[i,j] - dataMatrix[i,j])/abs(newDataMatrix[i,j]))* 100
                    if tempError > error:
                        error = tempError
                    if tempError > 100:
                        #         print("Cycle:",k,"OLD",dataMatrix[i,j],"NEW",newDataMatrix[i,j],"Error",
(abs(newDataMatrix[i,j] - dataMatrix[i,j])/abs(newDataMatrix[i,j]))* 100 )
            else:
                newDataMatrix[i,j] = dataMatrix[i,j]
    dataMatrix = newDataMatrix
    errors["cycle"].append(k)
    errors["value"].append(error)
    # print("Finish cycle ",k," with error: ",error)
```
````

Figure 4. A fragment of a Python program

Source: developed by the authors

Two matrices were defined for the calculations: a control matrix and a data matrix. The values of the elements of the control matrix (earthen dam on an impermeable base) are shown on Figure 5. The initial values of the elements of the data matrix (earthen dam on an impermeable base) are shown on Figure 6.

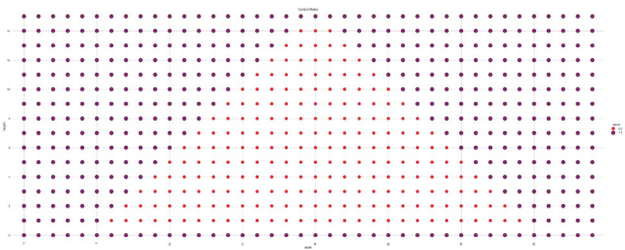

Figure 5. Values of the elements of the control matrix (earthen dam on an impermeable base)

Source: developed by the authors

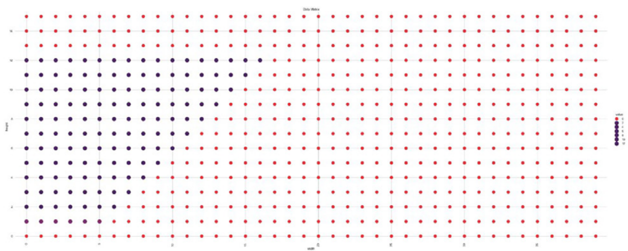

Figure 6. Values of data matrix elements (earthen dam on impermeable base)

Source: developed by the authors

For clarity, Figure 7 shows the convergence graph of the iterative process. The analysis of the graph allows concluding that after the 200[th] cycle, the values of the data matrix elements obtained as a result of iterative calculations hardly change, and the convergence process is short-lived.

The graph of the depression line for the earth dam on an impermeable base, which was obtained from the calculations, is shown on Figure 8. The analysis of the results suggests that the depression line does not extend to the downstream slope of the dam, and therefore no filtration deformation will occur.

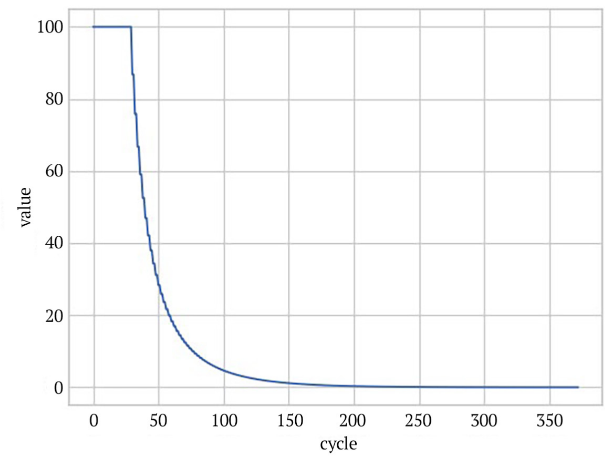

Figure 7. Convergence graph of the iterative process (earthen dam on an impermeable base)
Source: developed by the authors

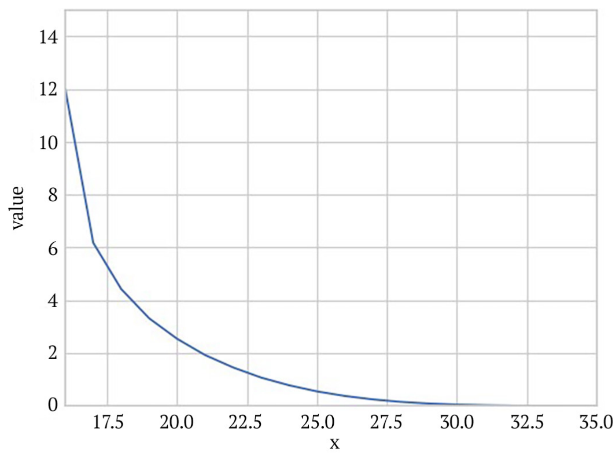

Figure 8. Depression line graph (earthen dam on impermeable foundation)
Source: developed by the authors

The proposed method of organizing the iterative computing process has several advantages that make it attractive for use:

1. Formalization and standardization of operations can simplify the process of developing software for hydraulic structures. Developers can concentrate on improving algorithms and models instead of spending time on implementation details, which increases the efficiency and speed of development.

2. The use of common loop statements makes the logic of the iteration process clear and easy to reproduce. This simplifies code debugging and allows developers to quickly implement and test new ideas.

3. The use of an additional array to determine the order of operations for each node provides flexibility in working with mesh configurations. This allows easily changing the grid boundaries and adjusting the calculation process to meet the specific requirements of the task.

4. The logical simplicity of the algorithm contributes to its successful adaptation for use in multiprocessor systems, which is important in the context of modern computing environments. This makes it possible to optimally use the power of multiprocessor systems and accelerate calculations, reducing the time for solving complex hydraulic engineering problems.

Thus, the proposed method of iterative calculations for finding the values of the potential function is not only effective but also quite versatile in its application to the design and modelling of hydraulic structures. It allows making the process of software development for hydraulic engineering facilities simpler and more structured, provides flexibility in working with mesh configurations and ensures easy adaptation for use in different computing environments, which is critical in hydraulic modelling. This method opens up wide possibilities for solving complex computational problems and studying various physical phenomena arising in hydraulic engineering.

DISCUSSION

In today's world, the role of hydraulic structures in providing water resources and controlling water flows is becoming increasingly important. Earthen structures such as dams, canals, and reservoirs play a crucial role in providing water for industry, agriculture and urban consumption, as well as regulating water flows to prevent floods and meet energy needs. However, earthworks also face numerous challenges, including seepage through their structures. Water infiltration through hydraulic structures can cause destruction and damage, threatening the quality and safe operation of these structures. In this context, the development of software for calculating seepage through earthen hydraulic structures is of great importance. Properly designed and calculated systems can effectively minimize seepage risks and ensure the safety and reliability of hydraulic structures. Such software will enable engineers and researchers to perform accurate and reliable seepage analyses of hydraulic structures, opening up new opportunities for improving their design, construction, and operation. Thus, mathematical modelling methods are an integral part of achieving accuracy and efficiency in predicting hydrodynamic phenomena (Sachaniuk-Kavets'ka *et al.*, 2022). They can be used to create analytical and numerical models that allow predicting the movement of water through hydraulic structures, taking into account various hydrodynamic processes occurring in the environment of earth structures. In addition, mathematical modelling methods allow for various sensitivity and statistical analyses to help understand the impact of various factors on the operation of hydraulic structures, as well as to predict and manage potential risks and emergencies (Afgan *et al.*, 2023). This approach is key to ensuring the safety and efficiency of hydraulic structures in different conditions and circumstances.

The work of X. Gao *et al.* (2023) used the finite element method to predict emergencies in hydraulic structures. By solving the filtration, the finite element method showed the ability to take into account the various physical processes occurring in the system. Also, a particular advantage of the finite element method was its ability to adapt to different conditions. It is noted that it can be used to model different types of hydraulic structures and in different geographical conditions. Such an approach can ensure not only the safety and stability of hydraulic structures, but also allow optimizing their efficiency while minimizing risks and costs. Comparing the results of this study with those of the researchers, it should be noted that both studies reflect significant progress in the use of mathematical modelling for forecasting emergencies at hydraulic structures, but apply different approaches and methods. In contrast to the work of the researchers using different mathematical models, including Laplace's equation and Darcy's law, this study developed and proposed an effective method that allows for versatility in applying it to different hydraulic structures and processes. In general, both approaches can be complemented and applied depending on specific needs and circumstances, providing a comprehensive and reliable modelling of hydraulic structures and their operation.

The work of S. Sun *et al.* (2020) also used the finite element method, which allowed for detailed modelling of hydraulic structures and their environment. This method allows for the consideration of complex geometric configurations of structures, material properties, and hydrodynamic conditions that affect the movement of fluid through the system. The use of the finite element method in the work of the researchers made it possible to study various aspects of filtration and pressure distribution in hydraulic structures, which allowed for accurate predictions and the development of effective measures to prevent possible emergencies. It allowed us to consider in detail the inhomogeneities and asymmetries in the geometry of hydraulic structures, as well as to take into account various hydrodynamic phenomena such as turbulence and heat transfer. It should be noted that in this study, similar to the work of the researchers also used the finite element method to model hydraulic structures and their environment. In the work of the researchers, the finite element method allowed for a detailed study of various aspects of filtration and pressure distribution in hydraulic structures, taking into account complex geometric configurations of structures, material properties, and hydrodynamic conditions. This has helped to develop effective measures to prevent possible emergencies and enabled accurate forecasts to be made. In this study, the use of the iterative computing method for modelling hydraulic engineering problems also yielded significant results. This method allowed for the efficient modelling of filtration flows and the consideration of various physical conditions such as turbulence and viscosity (Moldabayeva *et al.*, 2021). Although the proposed method is not as detailed in taking into account the geometric details of structures as the finite element method, it allows focusing on a wide range of hydraulic engineering problems and quickly developing effective solutions for them.

In the study by E. Mahiques *et al.* (2023), authors used the finite element method to perform mathematical modelling of filtration processes with great accuracy. This method, which is based on numerical calculations, made it possible to take into account complex geometric configurations, soil properties, and hydrodynamic conditions of fluid movement in the system. The use of the finite element method in the study by the researchers made it possible to conduct a detailed analysis of the impact of various factors on the filtration process. In particular, it was possible to investigate the influence of geometric parameters of hydraulic structures, such as shape and size, on pressure distribution and liquid penetration. Also, the influence of hydrodynamic conditions, such as the speed and direction of fluid movement, on the efficiency of filtration processes was taken into account. Analysing the results of the researchers, it should be noted that a particular advantage of the finite element method was the ability to conduct a detailed analysis of the impact of various factors on the filtration process, including the geometric parameters of hydraulic structures and hydrodynamic conditions. In turn, this study used the method of iterative calculations to find the values of the potential function in hydraulic engineering problems. Although this method is not as detailed in taking into account the geometric details of structures as the bounded element method, it is more versatile and adaptable to a wide range of hydraulic engineering problems. Taking into account the peculiarities of both approaches, it can be noted that the finite element method is detailed and effective for detailed analysis, while the iterative computation method is more general and can be applied to a wide range of hydraulic engineering problems (Cherniha *et al.*, 2016). The choice of method depends on the specific needs of the study and time and resource constraints.

In the work by M. Cremonesi *et al.* (2020), it was noted that the bounded element method has significant advantages over other approaches, as it allows for accurate consideration of the complex geometry and inhomogeneities in hydraulic structures that may occur in real conditions. In addition, it was found that the bounded element method provides high accuracy of the modelling results and allows analysing the impact of various factors on the filtration process. In particular, it made it possible to take into account variable hydraulic conditions, including different fluid flow rates and directions, as well as different boundary conditions at the boundaries of hydraulic structures. One of the key features of the finite element method, as noted by the researchers is its ability to accurately account for complex geometries and inhomogeneities in hydraulic structures. This makes it particularly useful for modelling real hydraulic structures where a variety of geometric features may occur. However, it is worth noting that the finite element method can be computationally intensive and time-consuming to perform calculations, especially in the case of large and complex hydraulic systems (Dyvak *et al.*, 2022). This may be a limitation for the use of this method in some cases where computational speed is of the essence. In addition, the successful application of the finite element method requires detailed data on the geometry and material properties of hydraulic structures, which can be a challenge in some situations.

In the work by M. Wu *et al.* (2020), the finite element method was used to model two-dimensional stationary filtration flows through earthen hydraulic structures. It is emphasized that the use of the finite element method made it possible to optimize the calculations and reduce the computational complexity of the model. This allowed for faster and more efficient results, which is important for practical applications in the field of hydraulic engineering. The results of the study showed that the finite element model allows for accurate reproduction of filtration processes in hydraulic structures of various configurations and sizes. Compared to the work of the researchers this study is aimed at developing a method of iterative calculations to find the values of the potential function in hydraulic engineering problems. Comparing these studies, it can be noted that both have their advantages. However, it is important to note that the present study is more focused on the iterative computation method, which is aimed at improving the known methods of numerical modelling, in particular for two-dimensional stationary filtration flows through hydraulic structures. This approach allows achieving accurate results with minimal computational complexity and contributes to more efficient use of computing resources. Thus, both approaches have their advantages and can be useful in different scenarios and for solving different problems in the field of hydraulic engineering.

In general, the research conducted within the framework of the analysis of software for calculating filtration through earthen hydraulic structures is an important contribution to the field of hydraulic engineering and hydrology. The results of these studies demonstrate a high level of efficiency and accuracy of numerical modelling, which allows for forecasting and developing measures to prevent possible emergencies at hydraulic structures. The use of different mathematical modelling methods underlines the importance of a multidisciplinary approach to solving problems in this area. Such studies provide valuable guidance for the development and improvement of software tools used to analyse and model hydraulic systems, ensuring the safety and stability of hydraulic structures and contributing to the optimization of their design and performance.

CONCLUSIONS

As a result of this study, software based on the method of iterative calculations was developed to find the values of the potential function in hydraulic engineering problems. This method proved to be effective and quite versatile in its application to various hydraulic structures and processes. The implementation of the proposed method can significantly simplify the process of developing software for the analysis and modelling of hydraulic systems. The proposed method makes it possible to implement software for diagnostic modelling of filtration flows in the body of earthen hydraulic structures. The main goal of the work was achieved through the use of various mathematical models, including the Laplace equation. The study also used such a law as Darcy's law, which describes the movement of fluid in porous media. Darcy's law was used to describe filtration processes in porous media and to determine the filtration coefficient, which characterizes the permeability of a liquid through a porous medium. This law is one of the main tools in hydrodynamics and hydraulic engineering for calculating fluid flows through soil, including problems related to water supply, drainage, and other hydraulic structures.

In general, the combination of Darcy's law, Laplace's equations, and other physical laws together with numerical approximation methods has allowed the development of efficient and accurate mathematical models for the analysis and modelling of various hydraulic systems and processes. Thus, formalization of operations and standardization of approaches can allow developers to focus on the algorithmic aspects of the problem, which speeds up the development process and increases productivity. The use of common loop statements makes the method understandable and accessible to developers. The logic of the iterative process becomes easy to understand, which contributes to the rapid development and maintenance of software. The use of an additional array to determine the order of operations for each node allows changing mesh configurations and adapting the method to various hydraulic engineering problems. This provides flexibility in dealing with different types of problems and allows for efficient solution of complex hydraulic engineering challenges. In general, the method of iterative calculations for finding the values of the potential function in hydraulic engineering problems opens up wide opportunities for solving complex computational problems and studying various physical phenomena in hydraulic engineering. It should also be noted that due to the implementation of the proposed iterative method, the accuracy of calculations sufficient for practical use was achieved.

Directions for further research in this area may include further refinement of the iterative computation method, extension of the application to a variety of hydraulic systems and conditions, and the possibility of combining this method with other technologies, opening up new perspectives in solving complex hydraulic problems.

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None.

CONFLICT OF INTEREST

None.

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Програмно-алгоритмічне забезпечення у складі регіональних систем моніторингу стану об'єктів для розрахунку фільтрації крізь земляні гідроспоруди

Анотація. Основою забезпечення безпечної експлуатації гідроспоруд є системи моніторингу стану об'єктів. Впровадження в склад програмного забезпечення таких систем програмних засобів для діагностичного моделювання фільтраційних течій – це об'єктивний крок, викликаний реальними подіями останнього часу. Метою даного дослідження є покращення точності вже існуючих методів для чисельного моделювання двовимірних стаціонарних фільтраційних течій за допомогою методу сіток у складі програмного забезпечення системи моніторингу стану об'єкту. Методи, що використовувалися в ході дослідження, включали в себе метод сіток, метод скінчених різниць, а також методи апроксимації та чисельні алгоритми. В результаті дослідження було встановлено, що запропонований спосіб організації ітераційного обчислювального процесу виявився ефективним та корисним. Його застосування дозволяє значно зменшити складність розробки програмного забезпечення, оскільки він базується на формалізації та стандартизації операцій, що спрощує процес програмування. Крім того, використання звичайних операторів циклу робить процес написання програм більш зрозумілим та доступним для розробників. Додатковий масив, який використовується в алгоритмі, дозволяє з легкістю змінювати конфігурацію границь сіткової області та порядок виконання операцій для кожного вузла, що робить метод гнучким та придатним для різноманітних викликів. Особливою перевагою алгоритму є його логічна простота, що сприяє успішній адаптації у випадку використання багатопроцесорних систем. Таким чином, результати дослідження підтверджують високу ефективність та потенціал запропонованого методу для використання у практичних обчислювальних завданнях. Запропонований метод ітераційних обчислень має значне практичне значення в галузі розробки програмного забезпечення для чисельного моделювання, адже його використання спрощує процес програмування та забезпечує гнучкість у роботі з різними умовами задач, що робить його важливим інструментом для широкого кола досліджень та практичних застосувань в гідротехніці та суміжних галузях

Ключові слова: системи моніторингу стану об'єкту; метод сіток; закон Дарсі; пористе середовище; принцип скінчених різниць; алгоритм ітераційних обчислень