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# Optimization of the joint startup of the boom and load hoisting mechanisms of a jib crane

**Abstract.** During the joint motion of the boom and load hoisting mechanisms, dynamic loads increase, leading to additional energy consumption, which subsequently contributes to the deterioration of the crane's structure. The aim of the research was to enhance the efficiency of the jib crane by optimising the joint startup modes of the boom and load hoisting mechanisms, which will minimise energy consumption. To achieve this aim, methods of analytical mechanics, variational calculus, and a modified particle swarm optimisation metaheuristic method were used. As a result of using these methods, the joint startup of the boom and load hoisting mechanisms was optimised. The joint motion of the crane mechanisms is represented by a dynamic model with four degrees of freedom, which accounts for the main motion of the mechanisms, as well as the elastic oscillations of the load hoisting mechanism's drive and low-frequency oscillations of the load on a flexible suspension. Based on the dynamic model, a mathematical model in the form of a system of second-order

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differential equations was constructed, which was then reduced to a system of two fourth-order equations. The synthesis of the optimal startup mode of the mechanisms was carried out according to the criterion of the root mean square value of the total power of the drives, taking into account the constraints on the driving torques of the drives. The constrained optimisation problem was reduced to an unconstrained optimisation problem by developing a generalised criterion. The nonlinear problem of optimising the joint startup mode for the boom and load hoisting mechanisms of the crane was solved using a modified particle swarm optimisation metaheuristic method. As a result of the optimisation, startup modes for the boom and load hoisting mechanisms were obtained that minimise the total power of the drives and eliminate low- and high-frequency oscillations of the crane's structural elements, which leads to increased reliability and reduced energy consumption. This mode is recommended for use in the control systems of crane boom and load hoisting mechanisms

**Keywords**: dynamic model; mathematical model; optimisation criterion; nonlinear optimisation problem; energy consumption

## INTRODUCTION

To increase the productivity of jib cranes, the movements of the boom and load hoisting mechanisms are combined. During the simultaneous startup of these mechanisms, increased dynamic loads and oscillations arise in the elements of the crane boom system. The greatest undesirable impact of these loads occurs when the boom and load hoisting mechanisms undergo simultaneous transient processes (startup, braking, or speed changes). In this case, spatial oscillations of the load on a flexible suspension increase, leading to higher energy consumption, decreased reliability of the crane boom system, and complications for both operating personnel and crane operators. As a result, the problem of selecting startup modes for the boom and load hoisting mechanisms arises, aiming to minimise dynamic forces and eliminate oscillations in the structural elements and the load on a flexible suspension. The effectiveness of jib cranes depends on the magnitude of the dynamic loads acting on the structural elements and the nature of their variation over time.

Considerable attention has been devoted to the study of the motion dynamics of overhead cranes, including jib cranes. The paper by J. Ye & J. Huang (2023) has been investigated the dynamics of cargo transportation by crane reloaders. In this paper, a dynamic model of a tower crane is developed. An equation for predicting coupled-system frequencies is then presented based on the analytical model. Additionally, a hybrid piecewise smoother is proposed to filter out coupled oscillations among jib deflection, load swing, and load twisting.The authors of the article M.M. Bello *et al*. (2024) present modelling and analysis of the dynamic characteristics of a double-pendulum overhead crane. The analysis of the dynamic characteristics of the crane can be useful for developing efficient controllers. The work by M. Michna *et al*. (2020) is dedicated to modelling the dynamics of the movement of a tower crane when testing the operation of individual crane mechanisms. The study of the motion dynamics of an overhead crane with a new wheel design is the subject of the work by N. Fidrovska *et al*. (2021), where stresses in the crane structure elements are determined. In the work by R. Miranda-Colorado (2021), an anti-sway observer for 2D crane systems during hoisting and lowering of loads has been developed. In the work by T. Yang *et al*. (2020), adaptive control against the swinging of a ship crane with roll movements has been implemented based on the developed neural network. The study by R. Buczkowski & B. Żyliński (2021) demonstrates that reducing dynamic loads and swinging of the load on a flexible suspension increases the reliability of jib cranes. Dynamic loads and swinging of the load on a flexible suspension also affect its positioning accuracy during loading, unloading, and assembly operations (Kovalenko *et al*., 2023). During the simultaneous operation of drive mechanisms, dynamic loads in the structural elements and the drives of jib cranes significantly increase. In the scientific work by O. Podoliak *et al*. (2021), modelling and investigation of the dynamics of the combined motion of the mechanisms for load hoisting, changing its outreach and rotation have been carried out. The dynamics of a double-jib crane with a nonlinear PID (proportional-integral-derivative) control system were investigated in a study by N. Sun *et al.* (2019).

The analysis of the conducted research shows that considerable attention has been devoted to studying the dynamics of overhead cranes and the methods of their control. Consequently, these issues remain relevant and require further investigation, particularly concerning the combined motion of the boom and load hoisting mechanisms.

Based on the above, this research aimed to enhance the efficiency of the crane's boom and load hoisting mechanisms by optimising the operational modes of the drive mechanisms. This optimisation minimises the impact of dynamic loads and energy consumption while also reducing high-frequency oscillations of the structural elements and pendulum oscillations of the load on a flexible suspension.

## MATERIALS AND METHODS

The joint operation of the boom and load hoisting mechanisms of a jib crane was represented by a discrete dynamic model (Fig. 1). In this model, the boom 1 was represented as an absolutely rigid link that rotated about the lower hinge with a moment of inertia $J_1$ relative to the rotation axis. It was driven by a hydraulic cylinder 2, which generated the torque $M_1$ about its axis of rotation. The hoisting of load 3, with the mass $m$, was carried out by a hoisting mechanism drive, which was reduced to the drive drum 4 with the moment of inertia of the model $J_2$. A rope with a stiffness coefficient $c$ of a single tackle 5 with $n$ multiplicity is wound onto drum 4 at one end and attached to the boom head at the other.

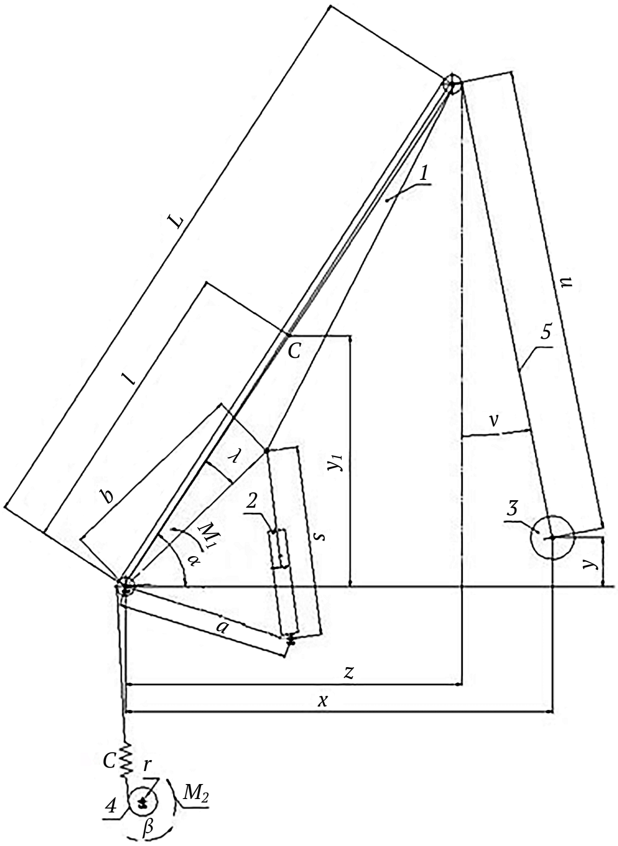


**Figure 1.** Dynamic model of a jib crane
**Note:** 1 – boom, 2 – hydraulic cylinder, 3 – load, 4 – drive drum, 5 – single tackle, $x$ – generalised coordinate of the load outreach, $z$ – horizontal coordinate of the boom outreach, $y$ – vertical coordinate of the load position, $y_1$ – vertical coordinate of the boom's centre of mass, $s$ – working length of the hydraulic cylinder, $b$, $\lambda$ – length of the strut from the lower hinge of the boom to the axis of the hydraulic cylinder's rod attachment and its inclination relative to the boom axis, $a$ – length of the strut from the lower hinge of the boom to the axis of the hydraulic cylinder's rod attachment and its inclination relative to the boom axis, $u$ – length of the flexible load suspension, $c$ – centre of mass of the boom, $\alpha$ – angular coordinate of the boom rotation, $\beta$ – angular coordinate of the drum rotation, $L$ – boom length, $l$ – position of the centre of mass of a boom relative to its lower hinge, $v$ angular coordinate of the deviation of the load on a flexible suspension from the vertical, $r$ – radius of the drive drum of the load hoisting mechanism, $M_1$ – driving torque of the boom hoisting mechanism, $M_2$ – driving torque of the load hoisting mechanism
**Source:** compiled by the authors

The provided dynamic model of the combined motion of the boom and load hoisting mechanisms was represented as a mechanical system with four degrees of freedom. The generalised coordinates of this system were taken as the angular coordinates of the boom rotation $\alpha$ and the drum rotation $\beta$, as well as the linear coordinates of the length of the flexible suspension of the load $u$ and its outreach $x$.

The angular coordinate of the load deviation from the vertical was determined by the following expression:

$$v = \frac{x - L\cos\alpha}{u}, \tag{1}$$

where $L$ – the boom length.

The vertical coordinates of the centres of mass of the boom and the load were also determined:

$$y_1 = l \cdot sin\alpha, \tag{2}$$

$$y = \mathrm{L} \cdot sin\alpha - u \cdot cosv, \tag{3}$$

where $l$ – the position of the centre of mass of a boom relative to its lower hinge.

To construct a mathematical model of the combined motion of the boom and load hoisting mechanisms, second-order Lagrangian equations were used:

$$\begin{gathered} \frac{d}{dt}\cdot\frac{\partial T}{\partial \dot\alpha} - \frac{\partial T}{\partial \alpha} = M_1 - \frac{\partial \Pi}{\partial \alpha},\ \frac{d}{dt}\cdot\frac{\partial T}{\partial \dot\beta} - \frac{\partial T}{\partial \beta} = M_2 - \frac{\partial \Pi}{\partial \beta}, \\ \frac{d}{dt}\cdot\frac{\partial T}{\partial \dot u} - \frac{\partial T}{\partial u} = -\frac{\partial \Pi}{\partial u},\ \frac{d}{dt}\cdot\frac{\partial T}{\partial \dot x} - \frac{\partial T}{\partial x} = -\frac{\partial \Pi}{\partial x}. \end{gathered} \tag{4}$$

Here $T$, $\Pi$ – the kinetic and potential energy, respectively. These were defined as follows:

$$T = \frac{1}{2}J_1\dot\alpha^2 + \frac{1}{2}J_2\dot\beta^2 + \frac{1}{2}m(\dot u^2 + \dot x^2), \tag{5}$$

$$\begin{gathered} \Pi = \frac{1}{2}c(\beta \cdot r - (u_0 - u)n)^2 + \\ +(m_1 l + mL)g \cdot \sin\alpha - m \cdot g \cdot u \cdot \cos v. \end{gathered} \tag{6}$$

Here $m_1$ – the mass of a boom; $g$ – the acceleration due to gravity; $u_0$ – the length of a flexible load suspension at the beginning of the movement; $v$ – the angular coordinate of a flexible load suspension's deviation from the vertical; $r$ – the radius of the drive drum of the load hoisting mechanism. Taking the derivatives of expressions (5) and (6), according to system (4), the following results were obtained:

$$\frac{dT}{d\alpha} = \frac{\partial T}{\partial \beta} = \frac{\partial T}{\partial u} = \frac{\partial T}{\partial x} = 0, \tag{7}$$

$$\frac{dT}{d\dot\alpha} = J_1\dot\alpha,\ \frac{dT}{d\dot\beta} = J_2\dot\beta,\ \frac{dT}{d\dot u} = m \cdot \dot u,\ \frac{dT}{d\dot x} = m \cdot \dot x, \tag{8}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot\alpha} = J_1\ddot\alpha,\ \frac{d}{dt}\frac{\partial T}{\partial \dot\phi} = J_2\ddot\beta,\ \frac{d}{dt}\cdot\frac{\partial T}{\partial \dot u} = m \cdot \ddot u,\ \frac{d}{dt}\cdot\frac{\partial T}{\partial \dot x} = m \cdot \ddot x, \tag{9}$$

$$\frac{\partial \Pi}{\partial \alpha} = (m_1 l + mL)g\cos\alpha + mgu\frac{\partial v}{\partial \alpha}\sin v, \tag{10}$$

$$\frac{\partial \Pi}{\partial \beta} = c \cdot r(\beta \cdot r - (u_0 - u)n), \tag{11}$$

$$\frac{\partial \Pi}{\partial u} = c \cdot n(\beta \cdot r - (u_0 - u)n) - mg(\cos v - u\frac{\partial v}{\partial u}\sin v), \tag{12}$$

$$\frac{\partial \Pi}{\partial x} = mgu\frac{\partial v}{\partial x}\sin v. \tag{13}$$

Taking into account that the angular deviation coordinate of the flexible suspension of the load $v$ did not exceed 12 degrees, it was assumed that $\sin v = v$ and $\cos v = 1$. Furthermore, the following substitution was made:

$$p = x - L\cos\alpha. \quad (14)$$

Based on the above and considering dependency (1) and its derivatives the generalised coordinates, expressions (10), (12), and (13) took the following forms:

$$\frac{\partial \Pi}{\partial \alpha} = ((m_1 l + mL)\cos\alpha + mL\frac{p}{u}\sin\alpha)g, \quad (15)$$

$$\frac{\partial \Pi}{\partial u} = cn(\beta r - (u_0 - u)n) - mg\left(\frac{p^2}{u^2} + 1\right), \quad (16)$$

$$\frac{\partial \Pi}{\partial x} = mg\frac{p}{u}. \quad (17)$$

As a result of substituting expressions (7), (9), (11), (14)-(17) into the system (4), the differential equations governing the combined motion of the boom and load hoisting mechanisms were obtained:

$$\begin{gathered} J_1\ddot{\alpha} = M_1 - ((m_1 l + mL)\cos\alpha + mL\frac{p}{u}\sin\alpha)g, \\ J_2\ddot{\beta} = M_2 - cr(\beta r - (u_0 - u)n), \\ m\ddot{u} = -cn(\beta r - (u_0 - u)n) - mg\left(\frac{p^2}{u^2} + 1\right), \\ \ddot{x} = -g\frac{p}{u}. \end{gathered} \quad (18)$$

When the boom and load hoisting mechanisms are started up simultaneously, dynamic loads in the structural elements and drive mechanisms increase, causing the load to swing on a flexible suspension. In addition, the crane's structural elements are worn, and energy consumption by the drives is also increasing. All of this leads to a decrease in the crane's operational efficiency. Therefore, there was a need to optimise the joint startup modes of the boom and load hoisting mechanisms to minimise energy consumption.

As the criterion for optimising the startup mode of the mechanisms, the root means square value of the total power of the boom and load hoisting drives during the startup process was used. The following dependency represented it:

$$P_{ck} = \left[\frac{1}{t_1}\int_0^{t_1}\{(M_1\dot{\alpha})^2 + (M_2\dot{\beta})^2\}\ dt\right]^{1/2} \to min. \quad (19)$$

The criterion (18), represented as an integral functional, needed to be minimised under the following boundary conditions of starting up the rotation mechanisms:

$$\begin{gathered} t = 0:\ \ \alpha = \alpha_0,\ \ \dot{\alpha} = 0,\ \ \beta = \frac{mg}{cnr}, \\ \dot{\beta} = 0,\ \ u = u_0,\ \ \dot{u} = 0,\ \ x = x_0,\ \ \dot{x} = 0, \end{gathered} \quad (20)$$

$$\begin{gathered} t = t_1:\ \alpha = \alpha_0 + \frac{\omega_1 t_1}{2},\ \dot{\alpha} = \omega_1,\ \beta = \frac{mg}{cnr} + \frac{\omega_2 t_1}{2}, \\ \dot{\beta} = \omega_2,\ u = u_0 - \frac{v_2 t_1}{2},\ \dot{u} = v_2,\ x = x_0 - \frac{v_1 t_1}{2},\ \dot{x} = v_1, \end{gathered} \quad (21)$$

where $t$ – the time coordinate; $t_1$ – the duration of the simultaneous startup of the boom and load hoisting mechanisms; $\omega_1$, $\omega_2$ – the steady-state angular velocities of the boom and the drive drum of the load hoisting mechanism; $\alpha_0$, $x_0$ – the initial positions of the boom and the load outreach, $v_1$, $v_2$ – steady-state velocities of the load outreach and load hoisting, respectively.

The constraints on the driving torques $M_1$ and $M_2$ of the boom and load hoisting mechanisms were defined by the following conditions:

$$M_{1max} \leqslant M_1 \leqslant M_{1max}, \quad (22)$$

$$M_{2max} \leqslant M_2 \leqslant M_{2max}, \quad (23)$$

where $M_{1min}$, $M_{1max}$ – the minimum and maximum allowable values of the drive torque of the boom hoisting mechanism; $M_{2min}$, $M_{2max}$ – the minimum and maximum allowable values of the drive torque of the load hoisting mechanism.

In the optimisation problem (18)-(23), it was necessary to find the modes of joint startup of the boom and load hoisting mechanisms of a jib crane that minimised the criterion (19) and satisfied the boundary conditions (20) and (21), as well as the constraints (22) and (23).

The criterion (19) was expressed as a function of the generalised coordinates of the boom hoisting $\alpha$ and the change in the load outreach $x$. To achieve this, the drive torques of the boom and load hoisting mechanisms were expressed using the first and second equations of the system (17):

$$M_1 = J_1\ddot{\alpha} + ((m_1 l + mL)\cos\alpha + mL\frac{p}{u}\sin\alpha)g, \quad (24)$$

$$M_2 = J_2\ddot{\beta} + cr(\beta r - (u_0 - u)n). \quad (25)$$

From the last equation of system (18), the deviation coordinate of the load from the vertical in the plane of the outreach change was expressed:

$$p = -\frac{u\cdot\ddot{x}}{g}. \quad (26)$$

Taking into account the expression (14), the following was obtained:

$$\cos\alpha = \frac{x-p}{L};\quad \sin\alpha = \sqrt{1 - [(x\ -\ p)/L]^2}. \quad (27)$$

From the first dependency in expressions (27), the generalized coordinate of the boom rotation was expressed:

$$\alpha = arccos\frac{(x-p)}{L}. \quad (28)$$

By taking time derivatives of expression (28), the angular velocity and acceleration of the boom were obtained:

$$\dot{\alpha} = -\frac{\dot{x}-\dot{p}}{L\cdot sin\alpha},\ \ddot{\alpha} = -\frac{1}{L}\cdot\frac{(\ddot{x}-\ddot{p})\sin\alpha-(\dot{x}-\dot{p})\dot{\alpha}\cos\alpha}{(\sin\alpha)^2}, \quad (29)$$

dependence (29) included time derivatives of expression (26), which were determined as follows:

$$\dot{p} = -\frac{\dot{u}\cdot\ddot{x}+u\cdot\dddot{x}}{g},\ \ddot{p} = -\frac{\ddot{u}\cdot\ddot{x}+2\cdot\dot{u}\cdot\dddot{x}+u\cdot\overset{IV}{x}}{g}. \quad (30)$$

additionally, from the penultimate equation of system (18), the angular coordinate of the drum drive mechanism for load hoisting was expressed as follows:

$$\beta = \frac{n}{r}(u_0 - u) + \frac{m}{c \cdot n \cdot r}\left[g\left(\frac{p^2}{u^2} + 1\right) - \ddot{u}\right]. \quad (31)$$

By taking the first- and second-time derivatives of equation (31), the angular velocity and acceleration of the drum drive mechanism for load hoisting were found:

$$\dot{\beta} = -\frac{n}{r}\dot{u} + \frac{m}{c \cdot n \cdot r}\left[2\frac{g}{u^3}p(u\dot{p} - \dot{u}p) - \dddot{u}\right], \quad (32)$$

$$\ddot{\beta} = -\frac{n}{r}\ddot{u} + \frac{m}{c \cdot n \cdot r}\left\{2\frac{g}{u^4}\left[\dot{p}\ \ (u \cdot \dot{p} - \dot{u} \cdot p) + p\ \ (u \cdot \ddot{p} - \ddot{u} \cdot p) - 3 \cdot \dot{u} \cdot p\ \ (u \cdot \dot{p} - \dot{u} \cdot p) - \overset{IV}{u}\right]\right\}. \quad (33)$$

As a result of substituting expressions (14) and (26)-(33) into the dependencies (24) and (25), the functions of the driving moments of the boom and load hoisting mechanisms were obtained. These functions depend on the linear generalised coordinates of the load outreach change $x$ and the length of the flexible load suspension $u$, as well as on their time derivatives.

To account for the constraints on the driving torques (22) and (23) in the stated optimisation problem, the following dimensionless generalized criterion was introduced and needed to be minimised:

$$Cr = \frac{P_{ck}}{P_{st}} + \delta_P(P_1 + P_2 + P_3 + P_4), \quad (34)$$

where $\delta_P$ – the penalty coefficient that accounts for the importance of satisfying the constraints (22) and (23); $P_{st}$ – the steady-state value of the total power of the drives of both mechanisms. To account for the constraints, penalty functions were constructed:

$$P_1 = \begin{cases} 0, if M_1 \geq M_{1\,min}; \\ \frac{M_1}{M_{1\,max}}, if\ M_1 < M_{1\,min}; \end{cases}$$

$$P_2 = \begin{cases} 0, if M_1 \leq M_{1\,max}; \\ \frac{M_1}{M_{1\,max}}, if\ M_1 > M_{1\,max}; \end{cases}$$

$$P_3 = \begin{cases} 0, if M_2 \geq M_{2\,min}; \\ \frac{M_2}{M_{2\,max}}, if\ M_2 < M_{2\,min}; \end{cases}$$

$$P_4 = \begin{cases} 0, if\ M_2 \leq M_{2\,max}; \\ \frac{M_2}{M_{2\,max}}, if\ M_2 > M_{2\,max}. \end{cases} \quad (35)$$

The value of the penalty coefficient $\delta_P = 10^8$ was chosen so that the optimisation algorithm first found a solution that satisfied all the constraints (22) and (23) (thereby reducing all the penalty functions to zero) and then found solutions that gradually reduced the integral functional (19).

From the expression (35), it was evident that all terms of the generalized criterion (34) were dimensionless. The first term corresponded to the value of the criterion (19) relative to the steady-state value of the total power of the drives of both mechanisms (i.e., when $t > t_1$), while the second term ensured compliance with the constraints (22) and (23).

The boundary conditions (20) and (21) were expressed in terms of the generalised coordinates of the length of the flexible load suspension and the change in its outreach $x$, and their time derivatives were determined.

$$t = 0:\ u = u_0,\ \dot{u} = 0,\ \ddot{u} = 0,\ \dddot{u} = 0,\ x = x_0,\ \dot{x} = 0,\ \ddot{x} = 0,\ \dddot{x} = 0, \quad (36)$$

$$t = t_1:\ u = u_0 - \frac{v^2 t_1}{2}, \dot{u} = v_2,\ \ddot{u} = 0,\ \dddot{u} = 0,$$
$$x = x_0 - \frac{v_1 t_1}{2},\ \dot{x} = v_1,\ \ddot{x} = 0,\ \dddot{x} = 0. \quad (37)$$

Here, $v_1$, $v_2$ – average steady-state rates of change of the load outreach and the length of its suspension, respectively.

The differential equations of motion (18), the integral functional (19), together with the constraints (22) and (23), expressions (24)-(35), and also the boundary conditions (36) and (37) represent an optimisation problem.

In this problem, it was necessary to determine the control action of the changes in the linear coordinates of the load outreach $x$ and the length of its suspension $u$, which minimised the value of the criterion (19) and ensured compliance with the constraints (22), (23), as well as the boundary conditions (36) and (37).

The presented optimisation problem was nonlinear, which required the use of an approximate method to solve it. The solutions to the optimisation problem were represented by the unknown functions $u(t)$ and $x(t)$ in the form of polynomials with two terms:

$$u(t) = u_1(t) + u_2(t), \quad (38)$$

$$x(t) = x_1(t) + x_2(t). \quad (39)$$

In the equations (38) and (39), the first terms $u_1(t)$ and $x_1(t)$ were the selected polynomials that satisfied the boundary conditions (36) and (37), while the second terms $u_2(t)$ and $x_2(t)$ were the polynomials that contained free unknown coefficients and provided the following zero boundary conditions:

$$u_2(0) = \dot{u}_2(0) = \ddot{u}(0) = \dddot{u}(0) = u_2(t_1) = \dot{u}_2(t_1) = \ddot{u}_2(t_1) = \dddot{u}_2(t_1) = 0, \quad (40)$$

$$x_2(0) = \dot{x}_2(0) = \ddot{x}_2(0) = \dddot{x}_2(0) = x_2(t_1) = \dot{x}_2(t_1) = \ddot{x}_2(t_1) = \dddot{x}_2(t_1) = 0. \quad (41)$$

The function $u_1(t)$ was represented as a seventh-order polynomial, which allowed for the satisfaction of the first part of the boundary conditions (36) and (37).

$$u_1(t) = \sum_{i=0}^{7} c_i t^i. \quad (42)$$

The function $x_1(t)$ was also represented as a seventh-order polynomial, which allowed for satisfying the second part of the boundary conditions (36) and (37).

$$x_1(t)=\sum_{k=0}^{7}c_k t^k. \quad (43)$$

In the boundary conditions (36) and (37), the time derivatives of the functions $u(t)$ and $x(t)$ were of the third order, that is why they were taken from the functions (42) and (43):

$$\dot{u}_1(t)=A_1+2A_2t+3A_3t^2+4A_4t^3+5A_5t^4+6A_6t^5+7A_7t^6, \quad (44)$$

$$\ddot{u}_1(t)=2A_2+6A_3t+12A_4t^2+20A_5t^3+30A_6t^4+42A_7t^5, \quad (45)$$

$$\dddot{u}_1(t)=6A_3+24A_4t+60A_5t^2+120A_6t^3+210A_7t^4, \quad (46)$$

$$W\dot{x}_1(t)=C_1+2C_2t+3C_3t^2+4C_4t^3+5C_5t_4+6C_6t^5+7C_7t^6, \quad (47)$$

$$\ddot{x}_1(t)=2C_2+6C_3t+12C_4t^2+20C_5t_3+30C_6t^4+42C_7t^5, \quad (48)$$

$$\dddot{x}_1(t)=6C_3+24C_4t+60C_5t^2+120C_6t^3+210C_7t^4. \quad (49)$$

Here, $A_0, A_1, \ldots, A_7, C_0, C_1, \ldots, C_7$ constants that are determined by the boundary conditions of the motion (36) and (37).

After substituting the boundary conditions (36) and (37) into the expressions (42) and (44)-(46), the following results were obtained:

$$A_0=u_0;\ A_1=0;\ A_2=0;\ A_3=0. \quad (50)$$

The constants $A_4$-$A_7$ were determined from the following system of linear equations:

$$\begin{aligned} A_4+A_5t_1+A_6t_1^2+A_7t_1^3 &= -\frac{v_2}{2t_1^3},\\ 4A_4+5A_5t_1+6A_6t_1^2+7A_7t_1^3 &= \frac{v_2}{t_1^3},\\ 6A_4+10A_5t_1+15A_6t_1^2+21A_7t_1^3 &= 0,\\ 4A_4+10A_5t_1+20A_6t_1^2+35A_7t_1^3 &= 0. \end{aligned} \quad (51)$$

After substituting the boundary conditions (36) and (37) into expressions (43) and (47)-(49), the following were obtained:

$$C_0=x_0;\ C_1=C_2=C_3=0. \quad (52)$$

The constants $C_4,\ldots,C_7$ were determined from the following system of linear algebraic equations:

$$\begin{aligned} C_4+C_5t_1+C_6t_1^2+C_7t_1^3 &= -\frac{v_1}{2t_1^3},\\ 4C_4+5C_5t_1+6C_6t_1^2+7C_7t_1^3 &= \frac{v_1}{t_1^3},\\ 6C_4+10C_5t_1+15C_6t_1^2+21C_7t_1^3 &= 0,\\ 4C_4+10C_5t_1+20C_6t_1^2+35C_7t_1^3 &= 0. \end{aligned} \quad (53)$$

As a result of substituting the constants from (50) and (52), as well as the solutions of the systems of linear algebraic equations (51) and (53), into expressions (42) and (43), the functions obtained, that satisfied the boundary conditions (36) and (37) for the coordinates $u$ and $x$ and their time derivatives.

The polynomials $u_2(t)$ and $x_2(t)$ represented by the following expressions:

$$u_2(t)=\left(\frac{t}{t_1}\right)^4\left(\frac{t_1-t}{t_1}\right)^4\sum_{j=0}^{p}B_j\left(\frac{t}{t_1}\right)^j\frac{v_2t_1}{2}, 0\le t\le t_1, \quad (54)$$

$$x_2(t)=\left(\frac{t}{t_1}\right)^4\left(\frac{t_1-t}{t_1}\right)^4\sum_{k=0}^{n}D_k\left(\frac{t}{t_1}\right)^k\frac{v_1t_1}{2}, 0\le t\le t_1, \quad (55)$$

where $B_0, B_1, \ldots, B_p, D_0, D_1, \ldots, D_n$ – free coefficients that determine the value of the optimization criterion (19); $\left(\frac{t}{t_1}\right)^4\left(\frac{t_1-t}{t_1}\right)^4$ – a multiplier that ensures the zero boundary conditions (40) and (41) are met at arbitrary values of the coefficients $B_0, B_1, \ldots, B_p$ and $D_0, D_1, \ldots, D_n$. These coefficients are free and ensure the determination of the minimum value of the criterion (19).

After substituting the expressions (46)-(54) into the dependencies (38) and (39), and taking into account the boundary conditions (40) and (41), the expressions for the functions $u(t)$ and $x(t)$ were obtained, which included the unknown free coefficients $B_0, B_1, \ldots, B_p$, and $D_0, D_1, \ldots, D_n$.

Using the expressions (28), (29), and (31)-(33), and considering the dependencies (26), (27), and (30), the angular coordinates of the boom rotation and the drive drum of the load hoisting mechanism, as well as their time derivatives were determined, these also depended on the free coefficients $B_0, B_1, \ldots, B_p, D_0, D_1 \ldots, D_n$. When integrating the criterion (19), it also became a function with the free coefficients $B_0, B_1, \ldots, B_p$, and $D_0, D_1, \ldots, D_n$. Thus, the approximate solution to the optimisation problem (18)-(23) was reduced to finding the minimum of the optimisation criterion (19) as a function with the many free coefficients $B_0, B_1, \ldots, B_p, D_0, D_1, \ldots, D_n$.

In order to solve the formulated optimisation problem, the Ring-Rot-PSO metaheuristic method developed by Y. Romasevych *et al*. (2022) was applied.

The number of iterations was set to 50, and the number of particles to 20. In this case, taking into account the expression (19) and the constraints (35), the generalized optimisation criterion (34) was expressed as a function of 10 free coefficients:

$$C_r=C_r(B_0, B_1, \ldots, B_4, D_0, D_1, \ldots, D_4). \quad (56)$$

Calculations for the optimal modes of joint startup of the boom and load hoisting mechanisms were performed based on the root mean square criterion of the total power of the drives (19) and the constraints on the driving moments (22) and (23), and with ensuring the boundary conditions (20) and (21) for the following values of the boom system parameters: $m=4500$ kg, $m_1=2700$ kg, $J_1=72900$ kg·m², $J_2=1183$ kg·m², $C=6.25\cdot10^6$ Nm/rad, $L=9.0$ m, $l=4.0$ m, $v_1=-0.3$ m/s, $\omega_1=0.0396$ rad/s, $v_2=-0.15$ m/s, $\omega_2=2.885$ rad/s, $u_0=8.0$ m, $n=4$, $t_1=5.0$ s, $g=9.81$ m/s², $M_{1\min}=0$, $M_{1\max}=130$ kNm, $M_{2min}=0$, $M_{2max}=4.0$ kNm, $\alpha_0=0.5857$ rad, $x_0=7.5$ m, $r=0.208$ m, $P_{st}=27000$ W.

## RESULTS AND DISCUSSION

As a result of solving the optimisation problem, the following values of the free coefficients were obtained: $B_0=6.9513$, $B_1=-5.11235$, $B_2=-13.0874$, $B_3=33.0285$, $D_0=-2.63782$, $D_1=0.829784$, $D_2=-9.80721$, $D_3=12.1441$. The results of applying the *VCP-PSO* method are shown in Figure 2.

Figure 2 shows that, during the operation of the optimisation algorithm, the first four iterations correspond to the penalty functions (35), which account for exceeding the maximum drive torques. Afterward, the algorithm finds a solution that satisfies one of the constraints (35), and during the subsequent iterations (from the fifth to the thirty-first), no significant improvements in terms of the value of the criterion *Cr* can be observed. However, on the thirty-second iteration, the *VCP-PSO* algorithm finds a solution that satisfies all the constraints (35). The subsequent iterations of the algorithm proceed in the region where all the constraints are satisfied, focusing solely on minimising the integral part (19) of the criterion $C_r$. Such a construction of the objective function (34) allows for a gradual approach to solving the problem: first ensuring the constraints (35), and then minimising the value of the integral functional (19).

Based on the obtained solution to the optimisation problem for the startup process of the boom and load hoisting mechanisms of a jib crane, the graphical dependencies of kinematic, dynamic, and energy characteristics have been built.

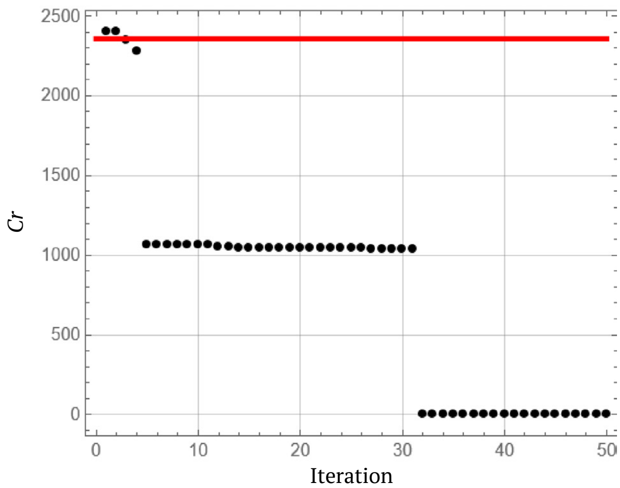


**Figure 2.** Plots of the decrease of the Cr criterion during the execution of the VCP-PSO algorithm iterations
**Source:** compiled by the authors

From Figure 3, can be seen that the angular velocity of the boom during startup, both in the baseline and in the optimal calculations, has a similar change pattern. It varies in an oscillatory mode. At the same time, the amplitude of velocity oscillations in the baseline calculation is slightly smaller. The oscillatory nature of the change in the angular velocity of the boom is caused by the nonlinear function of the boom position and the variable function of the static resistance forces acting on the boom.

At the same time, the angular velocity of the drive drum of the load hoisting mechanism changes smoothly during startup (Fig. 4). Such a change is observed both in the baseline and in the optimal calculations for the simultaneous motion of the boom and load hoisting mechanisms. The smooth change in the velocity of the drive drum is related to the fact that its position function is linear, while the static resistance force (the weight of the load) remains constant.

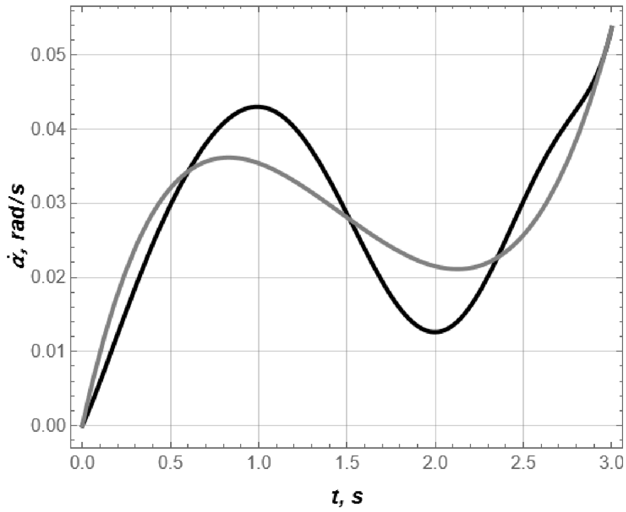


**Figure 3.** Plots of the angular velocity of boom rotation
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

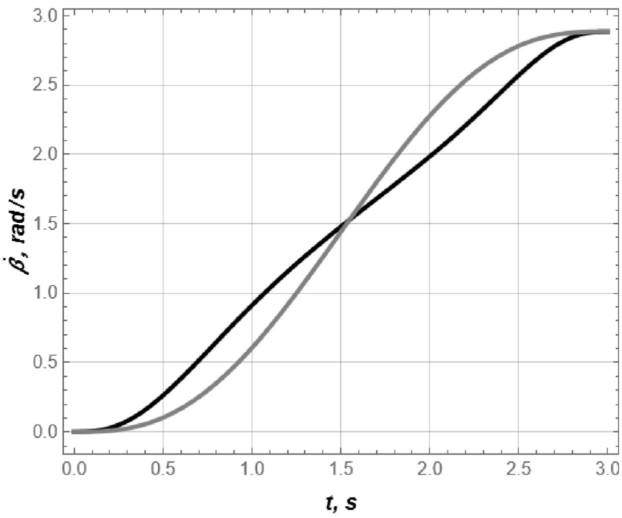


**Figure 4.** Plots of the angular velocity of the drive drum of a load hoisting mechanism
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

The phase portrait of rope oscillations of a load hoisting mechanism (Fig. 5) shows that the elastic oscillations of the rope damp out within one cycle of oscillation, both in the baseline and in the optimal calculations. However, in the optimal calculation, the deformation of the rope is slightly less and the maximum deformation rate is higher. Moreover, it should be noted that in the baseline calculation, the phase portrait of oscillations changes smoothly, which cannot be said about the phase portrait of oscillations in the optimal calculation, which has a more complex change. This is because the optimal calculation takes into account the actual loads on the structure and the existing constraints, whereas the baseline calculation only considers the boundary movement conditions of the system.

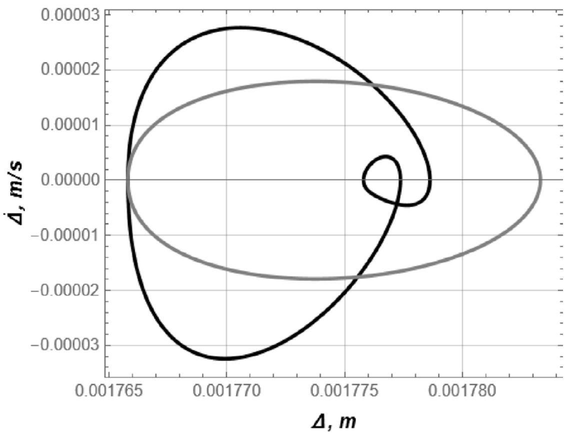


**Figure 5.** Phase portrait of elastic oscillations of the rope of a load hoisting mechanism
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

The phase portrait of the load oscillations on a flexible suspension (Fig. 6) shows that load oscillations damp out within one cycle, both in the baseline and optimal calculations of the system. However, in the optimal calculation, slightly larger maximum deviations from the vertical of the flexible suspension of the load are observed.

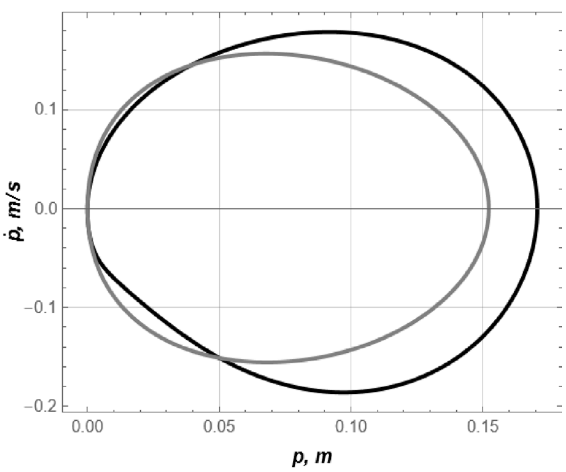


**Figure 6.** Phase portrait of pendulum oscillations of the load on a flexible suspension
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

In addition, to visualise the fulfilment of constraints (22)-(23) for the optimal solution to the problem, Figure 7 and 8 show the maximum constraints (dashed lines) of the drive torques of the mechanisms for boom and load hoisting. The drive torque of a boom hoisting mechanism (Fig. 7) decreases during startup from its maximum to a steady value, both in the baseline and optimal calculations. However, in the baseline calculation, the drive torque changes smoothly, while in the optimal calculation, it changes in an oscillatory manner. This is due to the fact that in the baseline calculation, there are no constraints on the permissible torque of 130 kNm, while in the optimal calculation, there is a constraint on the maximum value of the drive torque. A similar pattern is observed in the drive torque changes of the load hoisting mechanism (Fig. 8). In the baseline calculation, the torque changes smoothly but does not meet the established limit of 4000 Nm. At the same time, the optimal calculation ensures the torque constraint, but it changes with minor oscillations.

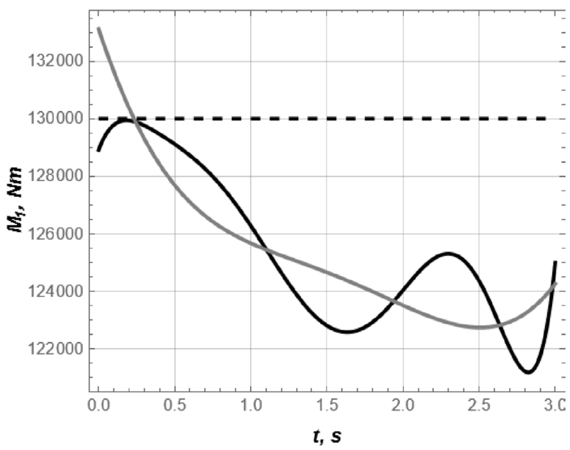


**Figure 7.** Plots of the drive torque of a boom hoisting mechanism
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

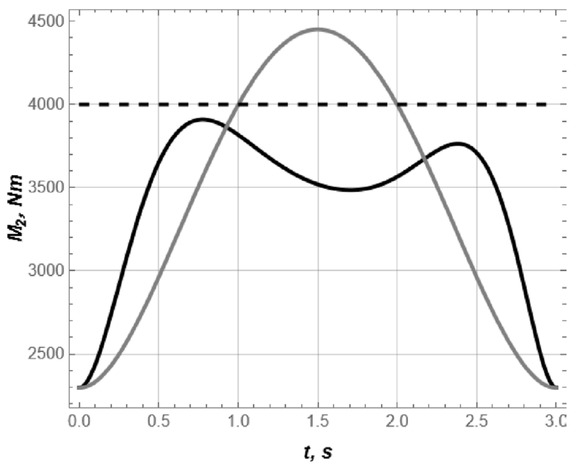


**Figure 8.** Plots of the drive torque of a load hoisting mechanism
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

The nature of the power change of the drive of a boom hoisting mechanism (Fig. 9) is similar in the baseline and optimal calculations. In both cases, oscillations are observed during the power change, but the amplitude of these oscillations is slightly higher in the optimal calculation compared to the baseline calculation.

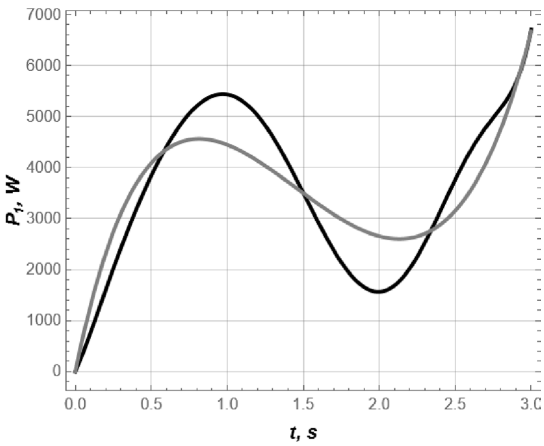


**Figure 9.** Plots of the power of the drive of a boom hoisting mechanism
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

The drive power of a load hoisting mechanism (Fig. 10) changes from zero to the steady-state value quite smoothly, both in the baseline and optimal calculations. At the same time, in the optimal calculation, the maximum power value slightly exceeds the same value in the baseline calculation.

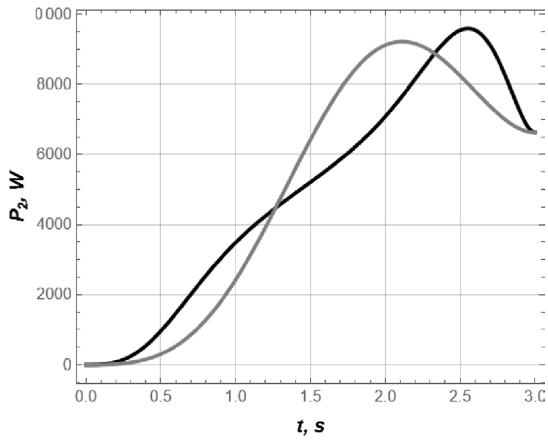


**Figure 10.** Plots of the drive power of a load hoisting mechanism
**Note:** the grey curves represent the baseline solution of the problem, and the black curves denote the optimal solution
**Source:** compiled by the authors

In addition, several numerical indicators are presented, corresponding to both solutions of the problem when determining the modes of combined motion for the boom and load hoisting mechanisms of a jib crane (Table 1).

**Table 1.** Numerical values of indicators that correspond to the baseline and optimal solutions of the problem (12)-(15)

| Evaluation indicator | Unit of measurement | The value of the indicator obtained for the solution | |
|---|---|---|---|
| | | baseline | optimal |
| Class of root mean square indicators | | | |
| Load deviation from the vertical | m | 0.09693 | 0.1008 |
| Elastic deformation of the rope of the load hoisting mechanism | m | 0.001775 | 0.001775 |
| Driving torque of the boom hoisting mechanism | kNm | 125.35 | 125.39 |
| Driving torque of the load hoisting mechanism | | 3.5263 | 3.4724 |
| Power of the boom hoisting mechanism | kW | 3.7015 | 3.8634 |
| Power of the load hoisting mechanism | | 6.0567 | 5.8577 |
| Class of maximum indicators | | | |
| Load deviation from the vertical | m | 0.1522 | 0.1706 |
| Elastic deformation of the rope of the load hoisting mechanism | m | 0.001783 | 0.001778 |
| Driving torque of the boom hoisting mechanism | kNm | 133.12 | 129.94 |
| Driving torque of the load hoisting mechanism | | 4.4509 | 3.9090 |
| Power of the boom hoisting mechanism | kW | 6.6636 | 6.7026 |
| Power of the load hoisting mechanism | | 9.2141 | 9.5834 |

**Source:** compiled by the authors

The analysis of Table 1 shows that the indicators for assessing the performance of a jib crane in the baseline and optimal calculations are generally quite close. At the same time, some of these indicators differ from each other in both directions. A more detailed analysis of the obtained indicators is conducted.

The load deviation from the vertical by the root mean square value is 3.8% lower in the baseline calculation than in the optimal one. However, the maximum value of this indicator in the baseline calculation is just 0.2% higher. The elastic deformation of the rope of the hoisting mechanism is the same in both calculations in terms of the root mean square value, and the maximum value of this indicator is 10.8% better in the baseline calculation. The driving torque of the boom hoisting mechanism in terms of the root mean square value, is almost the same in both the baseline and optimal calculations, at 125.4 kNm. However, the maximum value of this indicator is 2.4% lower in the optimal calculation. The driving torque of the load hoisting mechanism, both in terms of the root mean square and the maximum values, decreases by 1.6% and 13.9%, respectively, in the optimal calculation compared to the baseline calculation.

The power of the boom hoisting mechanism, in terms of the root mean square value and the maximum value, is 4.7% and 0.6% less, respectively, in the baseline calculation compared to the optimal calculation. The power of the load hoisting mechanism in terms of the root mean square value, is 3.4% lower and, in terms of the maximum value, is 4.0% higher in the optimal calculation than in the baseline calculation.

An additional analysis of scientific works has been conducted to compare the research results. Consequently, in the article by S.M. Fasih *et al.* (2020), the control of the load slewing mechanism is achieved using a neural network to generate the input data for a jib crane. In the work by A. Kostikov *et al*. (2019), control was implemented by manipulating the length of a flexible suspension to reduce load oscillations during jib crane movement. S. Tong *et al*. (2024) proposed a method for controlling the dynamic sliding mode and developed a dynamic model of a double-pendulum cable crane that considers wind disturbance. The authors of the article by Y. Qian *et al*. (2022) developed a method for controlling the movement of jib crane mechanisms. The work by F. Lui *et al.* (2021) carried out analysing the dynamic and vibration features of a tower crane. The results indicated that mass has little effect on the spatial swing angle, with lifting acceleration remaining stable between 0.004 m/s² and 0.01 m/s². In the work by R. Čápková & A. Kozáková (2019) considered identifying a dynamic model and designing a controller for changing the trolley position along the arm, as well as damping payload oscillation.

A considerable amount of work was devoted to optimisation of the parameters and motion modes of crane mechanisms has a significant impact on minimising dynamic loads. To solve these problems, metaheuristic optimisation methods by A.P. Piotrowski *et al*. (2020) have been widely applied. The main achievements of optimisation using swarm intelligence methods are presented in the works of other studies (Isiet & Gadala 2020; Houssein *et al*., 2021; Shami *et al*., 2022). The global optimisation of crane mechanism corrections has been investigated in the work by S. Chwastek (2023), based on the fundamental principles of mechanics. Also, a scientific article by S. Chwastek (2020) presented the optimised parameters of crane mechanisms, which make it possible to eliminate vibration (high-frequency oscillations) of the structural elements of a jib crane. At the same time, no restrictions were imposed on the driving forces of the drive mechanisms, meaning that the problem of unconstrained optimisation was solved. In the presented article, high-frequency oscillations of the load hoisting mechanism are also eliminated, but the constraints on the driving forces of drives of the boom and load hoisting mechanisms are introduced. Thus, the authors of this article have solved the problem of constrained optimisation, which is more general. Moreover, the authors of the presented article have developed their method for solving constrained optimisation problems for mechanisms and machines. In the paper by O. Podoliak *et al*. (2021), mathematical modelling of the dynamics of the combined motion of the mechanisms for load hoisting, changing the outreach and rotating a jib crane has been conducted. However, this work does not contain numerical calculations of the dynamics of the combined motion of the mechanisms under consideration. In the work by V. Loveikin *et al*. (2022), optimisation of the process of starting up the load outreach mechanism has been carried out, taking into account the oscillations of the load on a flexible suspension with a steady crane rotation. As a result of the optimisation, the maximum obtained deviation from the vertical of the flexible suspension is 0.031 rad, and the maximum speed is 0.036 rad/s. In the presented article, when changing the outreach and hoisting of the load, similar indicators are equal to 0.0213 rad and 0.0225 rad/s, respectively. The numerical indicators obtained in the presented article are quite close to those from previous studies. The slight difference between the indicators is because the considered variants of the mechanisms have different steady-state speeds of hoisting and changing the outreach of the load. The results presented in this article and in the articles of other authors lead to the conclusion that the research on the dynamics and optimisation of the motion modes of crane mechanisms is relevant. The results obtained in the article are close, and in some cases slightly better than the results obtained in similar studies.

## CONCLUSIONS

To minimise energy consumption in the drive mechanisms for boom and load hoisting of a jib crane with adhering the specified constraints, an optimisation problem has been formulated and solved. In this problem, the optimal startup modes of the boom and load hoisting mechanisms have been found by minimising an integral criterion that satisfies the boundary motion conditions of the system and the constraints on the drive torques of the mechanisms. Since the energy consumption of the boom system of a crane largely depends on the power of the drive mechanisms for boom and load hoisting, the criterion for optimisation has been chosen as the root mean square value of the total power of both drive mechanisms during the combined startup mode. To reduce the loads on the structural elements of a crane and the drive mechanisms, the minimum and maximum values of the drive torques have been considered as constraints. In the formulated optimisation problem, the criterion in the form of the root mean square value of the drive powers, along with the constraints on their drive torques, is expressed as a generalised optimisation criterion. In this problem, the issue of ensuring constraints on the drive torques has been addressed first, and then the problem of minimising the root mean square value of the criterion has been solved. Thus, the constrained optimisation problem has been reduced to an unconstrained optimisation problem.

Since the optimisation problem of finding the operating mode of the joint motion of the boom and load hoisting mechanisms of a crane is nonlinear, it could not be solved analytically. Instead, an approximate method has been applied. The solution to the problem has been sought on the class of polynomial functions. The sought functions of the boom and load hoisting mechanisms are represented as a sum of two polynomials. The first polynomial satisfies the boundary conditions of the motion, and the second includes free coefficients that minimise the generalised optimisation criterion. In the solved optimisation problem, the free coefficients have been determined using a modified metaheuristic method *VCP-PSO*.

The solution to the optimisation problem has made it possible to find the modes of joint startup of the boom and load hoisting mechanisms of a jib crane that minimise the total energy consumption of the drives. These results have been achieved by reducing the maximum value of the driving torque of the load hoisting mechanism by 13.9%.

In the process of optimising the modes of joint startup of the boom and load hoisting mechanisms, two calculation variants have been considered: the baseline one and the optimal one. These are compared with each other and show quite close results in almost all the indicators. It should be noted that the baseline calculation variant provides only boundary conditions of the motion, and the optimal one, in addition to boundary conditions, also provides constraints on the driving forces. Therefore, when choosing a movement mode, priority should be given to the optimal calculation method and the results obtained using this method. In further research on optimising the operating modes of a crane's boom system, it is necessary to consider a dynamic model with variable boom length.

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## CONFLICT OF INTEREST

The authors declare no conflict of interest.

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# Оптимізація сумісного пуску механізмів підйому стріли та вантажу стрілового крана

**Анотація.** В процесі сумісного руху механізмів підйому стріли та вантажу зростають динамічні навантаження, які приводять до додаткових витрат енергії, що в подальшому витрачається на руйнування конструкції крана. Мета досліджень полягає в підвищенні ефективності роботи стрілового крану шляхом оптимізації режимів сумісного пуску механізмів підйому стріли та вантажу, яка приводить до мінімізації енергетичних витрат. Для досягнення поставленої мети використані методи аналітичної механіки, варіаційного числення та модифікований метаевристичний метод рою часточок. В результаті використання наведених методів оптимізовано сумісний пуск механізмів підйому стріли та вантажу. Спільний рух механізмів крана представлено динамічною моделлю з чотирма ступенями вільності, де враховано основний рух механізмів, а також пружні коливання приводу механізму підйому вантажу та низькочастотні коливання вантажу на гнучкому підвісі. На основі динамічної моделі побудовано математичну модель у вигляді системи диференціальних рівнянь другого порядку, яку зведено до системи двох рівнянь четвертого порядку. Синтез оптимального режиму пуску механізмів здійснено за критерієм середньоквадратичного значення сумарної потужності приводів з врахуванням обмежень на рушійні моменти приводів. Поставлена задача умовної оптимізації зведена до задачі безумовної оптимізації шляхом розробки узагальненого критерію. Нелінійну задачу оптимізації режиму спільного пуску кранових механізмів підйому стріли та вантажу розв'язано модифікованим метаевристичним методом рою часточок. В результаті проведеної оптимізації отримано режими пуску механізмів підйому стріли та вантажу, які мінімізують сумарну потужність приводів і усувають низько та високочастотні коливання елементів конструкції крана. Це приводить до підвищення його надійності та зменшення енергетичних витрат. Такий режим рекомендовано використовувати у системах керування крановими механізмами підйому стріли та вантажу

**Ключові слова**: динамічна модель; математична модель; критерій оптимізації; нелінійна задача оптимізації; енергетичні витрати