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Optimisation of tower crane rotation mechanism force acceleration mode at steady payload hoisting

Abstract. The process of optimisation of the force acceleration mode of the rotation mechanism of the tower crane with the beam jib at steady payload hoisting was examined in the presented paper. The crane rotation mechanism was presented as a dynamic model with 4DOF in which the main motion of the rotation and payload hoisting mechanisms is considered as well as oscillations of the rotation part of the crane and the payload in the plane of slew. For such a dynamic model, a mathematical model which is a system of ordinary second order differential equations was formulated. The system of equations was reduced to the angular coordinate of the payload slew and was represented by the sixth order equation. The optimisation of the rotation mechanism's acceleration mode at steady payload hoisting was carried out based on the RMS of the drive torque criterion considering constraints on the drive torque and drive power. The optimisation criterion as well as the constraints were reduced to the generalised criterion. When optimising , the movements and velocities of the generalised rotation mechanism coordinates in the beginning and in the end of the acceleration process were chosen as motion boundary conditions. The acceleration mode of the crane rotation mechanism at steady payload hoisting was obtained as the result of optimisation. The obtained mode minimised the action of the dynamic payloads and decreases the maximum values of the crane's mast deformation, drive power, and deviation of the payload cable from the vertical. At that, the crane's elements' and payload's oscillations upon reaching the steady motion mode were eliminated. The results

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of the study may be used in automation systems for controlling the rotation mechanisms of tower cranes, which ensures improved efficiency and stability of work during payloads hoisting

Keywords: jib crane; hoisting and rotation mechanisms; dynamic payloads; optimal control; payload oscillation

INTRODUCTION

A combination of the rotation and payload hoisting mechanisms is performed to increase the capacity of tower cranes. During such motion of mechanisms, significant dynamic payloads occur in the elements of the crane and in drive mechanisms, along with oscillations of the payload on elastic suspension. These payloads exert the greatest undesirable influence when the rotation mechanism carries out the processes of acceleration or braking. During these processes, spatial oscillations of the payload on elastic suspension sharply increase. They influence energy losses, crane's work reliability and complicate the work of the staff and the crane operator. Determining an acceleration mode of the rotation mechanism at steady payload hoisting that decreases dynamic payloads and eliminates oscillations of the payload on elastic suspension is an important problem.

The efficiency of cranes work, tower cranes in particular, is determined based on their capacity, reliability, and safe exploitation. In work M. Michna *et al.* (2020) it was established that a minute decrease of dynamic payloads during the undergoing of transient processes significantly enhances the reliability indicators of cranes. Apart from that, dynamic payloads exert a significant influence on the accuracy of the positioning of the payload (Chwastek, 2021). Dynamic payloads increase significantly in crane structures at combining the work of drive mechanisms. At that, the swinging of the payload on the elastic suspension also increases.

The decrease of the dynamic payloads can be achieved by using fundamental principles of mechanics, based on which the methods of optimisation of motion of the crane mechanisms are built. In study by S. Chwastek (2020), criteria of optimisation as quadratic functionals for each mechanism of the tower crane were formulated based on the built mathematical model. The minimisation of the functionals allowed for obtaining a set of optimal parameters for crane mechanisms at which their interaction had to be effective. An innovative feature of the presented solution was the complex approach to the optimisation of the crane mechanisms with separate drives. This approach was focused on decreasing the oscillations of the payload on elastic suspension and minimising the forces that act in the haulage elements (cables). Much attention was given to the problems of optimisation of the parameters of the crane mechanisms and securing the reliability and safety of handling processes. At that, various programs and methods of study were employed, metaheuristic methods in optimisation problems for crane mechanisms in particular P. Adam *et al.* (2020).

The work by M. Ambrosino *et al.* (2020) established that productivity depends on the magnitude of dynamic payloads in the crane elements. The authors of the article O. Podolyak *et al.* (2022) noted the presence of dynamic payloads in the crane structure, which depend on the oscillations of the payload on the flexible suspension. Here, a mathematical model is presented that describes the processes of starting and braking of a crane and its dynamic payloads are determined. In the work by Q. Wu *et al.* (2020), a mathematical model is built and the dynamics of the movement of a crane with a double pendulum and with a distributed mass of the bridge girder are analysed.

The choice of motion modes for drive mechanisms that decrease the oscillations of the elements and increase the efficiency of cranes is quite a relevant problem and requires more comprehensive research.

The research purpose lied in increasing the efficiency of joint work of the tower crane rotation and payload hoisting mechanisms through optimising the acceleration mode of the rotation mechanism at steady operation of the payload hoisting mechanism.

MATERIALS AND METHODS

The jib system with a beam jib of the tower crane at joint rotation and payload hoisting mechanisms may be presented as a discrete dynamic model (Fig. 1). Such model comprises brought to the crane slew axis absolutely solid masses of the drive rotation mechanism with the inertia moment J_1 and the rotation part of the crane with the inertia moment J_2 that are connected together by an elastic element with the stiffness coefficient C. A trolley with the mass m_2 to the centre of gravity of which with the help of a variable length elastic suspension a payload with the mass m is attached. It is located on a beam jib in a stationary position. Drive torque M and resistance forces torque M_0 that are brought to the crane slew axis act in the drive rotation mechanism. The main motion of the drive mechanisms, elastic displacements of the rotation mechanism and oscillations of the payload on elastic suspension in the plane of the crane slew are considered in the presented dynamic model. Deviations of the elastic payload suspension from the vertical in the plane of the trolley movement were not considered, as they depend on the magnitude of centrifugal force that acts on the payload rather than on the crane rotation mode. Thus, the provided dynamic model of the jib system has 4DOF.

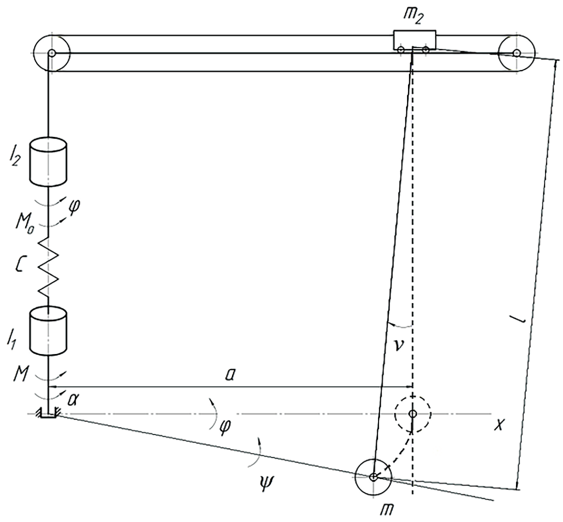

Figure 1. Dynamic model of the tower crane slew at steady payload hoisting

Note: J_1 – drive rotation mechanism with the inertia moment; J_2 – rotation part of the crane with the inertia moment; m_2 – trolley with the mass; m – payload with the mass; C – stiffness coefficient; M – drive torque; M_0 – resistance forces torque; l – length of the elastic suspension; a – payload outreach (parameter); v – angular coordinate of deviation of a flexible payload suspension from the vertical; α – generalised coordinate of the rotation mechanism drive; φ – generalised coordinate rotation part of the crane; ψ – generalised coordinate slew of the payload

Source: compiled by the authors

At that, the coordinate of the payload hoist (the length of the elastic suspension) changes with a constant rate v. It is determined by the dependency:

$$l=l_0+vt, \tag{1}$$

where l_0 – the initial length of the elastic payload suspension; t – time; v – the constant velocity of the payload hoist.

It was chosen angular coordinates of the brought mass of the rotation mechanism drive α, rotation part of the crane φ, and slew of the payload ψ as generalised coordinates of the dynamic model of the jib system at the rotation mechanism operation. At the same time, the trolley's position is unchanged, it is determined by the parameter a. The angular coordinate of the elastic payload suspension deviation from the vertical was determined by the following dependency:

$$v=\frac{a(\phi-\psi)}{l_0+vt}. \tag{2}$$

The second order Lagrange equations were used to build a mathematical model of the crane rotation mechanism motion at steady payload hoisting:

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\alpha}}-\frac{\partial T}{\partial\alpha}=\mathrm{M}\text{-}\frac{\partial\Pi}{\partial\alpha};\ \frac{d}{dt}\frac{\partial T}{\partial\dot{\phi}}-\frac{\partial T}{\partial\phi}=M_0-\frac{\partial\Pi}{\partial\phi};$$
$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\psi}}-\frac{\partial T}{\partial\psi}=-\frac{\partial\Pi}{\partial\psi}, \tag{3}$$

where, T and Π are the kinetic and potential energy, respectively, expressed as:

$$T=\frac{1}{2}J_1\dot{\alpha}^2+\frac{1}{2}(J_2+m_2a^2)\dot{\phi}^2+\frac{1}{2}m\left(a^2\dot{\psi}^2+v^2\right), \tag{4}$$

$$\Pi=\frac{1}{2}C(\alpha-\phi)^2-\mathrm{mglcos}v=\frac{1}{2}C(\alpha-\phi)^2-\mathrm{mg}\,(l_0+vt)\cos\frac{a(\phi-\psi)}{l_0+vt}. \tag{5}$$

Taking derivatives of expressions (4) and (5) according to system (3), the following results were obtained:

$$\frac{\partial T}{\partial\alpha}=\frac{\partial T}{\partial\phi}=\frac{\partial T}{\partial\psi}=0;\ \frac{\partial T}{\partial\dot{\alpha}}=J_1\dot{\alpha};$$
$$\frac{\partial T}{\partial\dot{\phi}}=(J_2+m_2a^2)\dot{\phi};\ \frac{\partial T}{\partial\dot{\psi}}=ma^2\dot{\psi}; \tag{6}$$

$$\frac{d}{\mathrm{d}t}\frac{\partial T}{\partial\dot{\alpha}}=J_1\ddot{\alpha};\ \frac{d}{\mathrm{d}t}\frac{\partial T}{\partial\dot{\phi}}=(J_2+m_2a^2)\ddot{\phi};$$
$$\frac{d}{\mathrm{d}t}\frac{\partial T}{\partial\dot{\psi}}=ma^2\ddot{\psi}; \tag{7}$$

$$\frac{\partial\Pi}{\partial\alpha}=C(\alpha-\phi); \tag{8}$$

$$\frac{\partial\Pi}{\partial\phi}=-C(\alpha-\phi)+m\cdot g\cdot a\cdot\sin\frac{a(\phi-\psi)}{l_0+vt}=$$
$$-C(\alpha-\phi)+\frac{\mathrm{mg}a^2}{l_0+vt}(\phi-\psi); \tag{9}$$

$$\frac{\partial\Pi}{\partial\psi}=-m\cdot g\cdot a\cdot\sin\frac{a(\phi-\psi)}{l_0+vt}=-\frac{\mathrm{mg}a^2}{l_0+vt}(\phi-\psi). \tag{10}$$

After substitution of expressions (6), ... (10) into the system (3) the equations of the tower crane rotation mechanism motion at steady payload hoisting the following expression was obtained:

$$J_1\ddot{\alpha}=\mathrm{M}\text{-}\mathrm{C}(\alpha-\phi);$$
$$(J_2+m_2a^2)\phi=-M_0+C(\alpha-\phi)\mathrm{ma}^2\frac{g}{l_0+vt}(\phi-\psi);$$
$$\ddot{\psi}=\frac{g}{l_0+vt}(\phi-\psi). \tag{11}$$

Significant dynamic payloads that lead to swinging of the payload on elastic suspension appear in the elements of the structure and drive mechanism in the rotation mechanism acceleration process, which decreases the reliability of the crane operation and increases energy losses. Thus, the need for optimising the motion mode of the drive rotation mechanism of the crane which would decrease the action of the dynamic payloads to minimum has appeared. Dynamic payloads largely depend on the magnitude and nature of the torque change of the drive mechanism. Thus, it was chosen the RMS (Root Mean Square) value of the drive torque during acceleration as a criterion of optimisation of the rotation mechanism acceleration mode at steady payload hoisting. It was determined by the following dependency:

$$M_{ck} = \left[\frac{1}{t_1}\int_0^{t_1} M^2\, dt\right]^{1/2} \rightarrow min. \qquad (12)$$

Criterion (12), which is an integral functional, must be minimised when meeting the following boundary conditions of the rotation mechanism motion:

$$t=0:\ \alpha=\frac{M_0}{C},\ \dot{\alpha}=0, \phi=0,\ \dot{\phi}=0, \psi=0,\ \dot{\psi}=0;$$
$$t=t_1:\ \alpha=\frac{M_0}{C}+\frac{\omega t_1}{2},\ \dot{\alpha}=\omega,\ \phi=\frac{\omega t_1}{2},\ \dot{\phi}=\omega, \psi=\frac{\omega t_1}{2},\ \dot{\psi}=\omega; \qquad (13)$$

and constraints on the drive torque M and the power of rotation mechanism drive P:

$$M_{\min} \leqslant M \leqslant M_{\max}; \qquad (14)$$
$$P_{\min} \leqslant P \leqslant P_{\max}. \qquad (15)$$

where t_1 – the duration of the rotation mechanism acceleration process; ω – the steady angular velocity of the rotation part of the crane; $M_{\min}$ and $M_{\max}$ – the minimum and maximum permissible values of the rotation mechanism drive torque; $P_{\min}$ and $P_{\max}$ – the minimum and maximum permissible values of the rotation mechanism drive power.

It was necessary to determine a tower crane rotation mechanism motion mode that ensures an extreme value of criterion (12) and meets boundary conditions (13) and constraints (14) and (15) in optimisation problem (12), ..., (15).

The criterion (12) was expressed using the generalised coordinate of the payload slew ψ and its derivatives with respect to time. For this, the expression for the drive torque from the first equation of system (11) was found:

$$M=J_1\ddot{\alpha}+C(\alpha-\phi). \qquad (16)$$

The expression of the angular coordinate of the rotation part of the crane from the last equation of system (11) was determined. In further it was expressed by using the payload slew coordinate and its second derivative with respect to time:

$$\phi = \psi + \frac{l_0+vt}{g}\ddot{\psi}. \qquad (17)$$

The angular velocity and acceleration of the rotation part of the crane by taking the first and the second derivatives with respect to time of expression (17) were determined:

$$\dot{\phi} = \dot{\psi} + \frac{v}{g}\ddot{\psi} + \frac{l_0+vt}{g}\dddot{\psi}. \qquad (18)$$

$$\ddot{\phi} = \ddot{\psi} + 2\frac{v}{g}\dddot{\psi} + \frac{l_0+vt}{g}\psi^{\mathrm{IV}}. \qquad (19)$$

From the second equation, considering the third equation of system (11), the last expression that belongs to dependency (16) was determined:

$$C(\alpha-\phi)=(J_2+m_2a^2)\ddot{\phi}+\mathrm{m}a^2\ddot{\psi}+M_0.$$

After substitution of expression (19) into the obtained dependency, the following expression may be obtained:

$$C(\alpha-\phi)=(J_2+m_2a^2)\left[\ddot{\psi}+2\frac{v}{g}\dddot{\psi}+\frac{l_0+vt}{g}\psi^{\mathrm{IV}}\right]+\mathrm{m}a^2\ddot{\psi}+M_0. \qquad (20)$$

From equation (20) the angular coordinate of the drive brought to the axis of the crane slew was expressed:

$$\alpha = \psi + \left[\frac{l_0+vt}{g}+\frac{J_2+(m+m_2)a^2}{C}\right]\ddot{\psi} + \frac{J_2+m_2a^2}{C}\left[2\frac{v}{g}\dddot{\psi}+\frac{l_0+vt}{g}\psi^{\mathrm{IV}}\right]+\frac{M_0}{C}. \qquad (21)$$

The first and the second derivatives with respect to time of expression (21) were obtained, the angular velocity and acceleration of the rotation mechanism drive that are reduced to the rotation part of the crane were determined:

$$\dot{\alpha} = \dot{\psi} + \frac{v}{g}\ddot{\psi} + \left[\frac{l_0+vt}{g}+\frac{J_2+(m+m_2)a^2}{C}\right]\dddot{\psi} + \frac{J_2+m_2a^2}{C}\left[3\frac{v}{g}\psi^{\mathrm{IV}}+\frac{l_0+vt}{g}\psi^{V}\right]; \qquad (22)$$

$$\ddot{\alpha} = \ddot{\psi} + 2\frac{v}{g}\dddot{\psi} + \left[\frac{l_0+vt}{g}+\frac{J_2+(m+m_2)a^2}{C}\right]\psi^{IV} + \frac{J_2+m_2a^2}{C}\left[4\frac{v}{g}\psi^{V}+\frac{l_0+vt}{g}\psi^{\mathrm{VI}}\right]. \qquad (23)$$

As a result of substitution expressions (20) and (23) into dependency (14) the function of the drive torque as the function of the angular coordinate of the payload slew and its derivatives with respect to time was obtained:

$$M = M_0 + [J_1+J_2+(m_2+m)a^2]\ddot{\psi} + 2(J_1+J_2+m_2a^2)\frac{v}{g}\dddot{\psi} + \left\{J_1\left[\frac{l_0+vt}{g}+\frac{J_2+(m+m_2)a^2}{C}\right]+(J_2+m_2a^2)\frac{l_0+vt}{g}\right\}\psi^{IV} + \frac{J_2+m_2a^2}{C}\left[4\frac{v}{g}\psi^{V}+\frac{l_0+vt}{g}\psi^{\mathrm{VI}}\right]. \qquad (24)$$

In order to consider constraints (14) and (15) in the stated optimisation problem the objective function to minimise was designed:

$$\mathrm{Cr} = M_{\mathrm{ck}} + I min, P_{max}P_{min}M_{max_{M_{min}}}$$

$$I\begin{cases}0, if\ \min(M) \geq M_{\min};\\ min(M)\ \delta_M,\ if \min(M) < M_{\min};\end{cases}_{M_{min}}$$

$$I\begin{cases}0, if\ \max(M) \geq M_{\min};\\ max(M)\ \delta_M,\ if \max(M) < M_{\max};\end{cases}_{M_{max}} \qquad (25)$$

$$I\begin{cases}0, if\ \min(P) \geq P_{\min};\\ min(P)\ \delta_P,\ if \min(P) < P_{\min};\end{cases}_{P_{min}}$$

$$I\begin{cases}0, if\ \max(P) \geq P_{\min};\\ max(P)\ \delta_P,\ if \max(P) < P_{\max};\end{cases}_{P_{max}}$$

where I_{Mmin} and I_{Mmax} the penalty components of the criterion that regulate the compliance with the first and the second inequalities of constraints (14), respectively; I_{Pmin} and I_{Pmax} are the penalty components of the criterion that correspond to the compliance with the first and the second inequalities of constraints (15), respectively; δ_M and δ_P are the weight coefficients that determine the magnitude of the penalties for non-compliance with constraints (14) and (15), respectively (within this study, it was assumed that $\delta_M = 10^{10}$ and $\delta_P = 10^8$).

The boundary conditions (13) were reduced to the generalised coordinate of the payload slew ψ and its derivatives with respect to time:

$$\begin{gathered} t=0: \psi=0, \dot{\psi}=0;\ \ddot{\psi}=0,\ \dddot{\psi}=0,\ \psi^{IV}=0,\ \psi^{V}=0; \\ t=t_1: \psi=\left(\frac{\omega t_1}{2}\right), \dot{\psi}=\omega;\ \ddot{\psi}=0,\ \dddot{\psi}=0,\ \psi^{IV}=0,\ \psi^{V}=0. \end{gathered} \tag{26}$$

where, ω – the steady angular velocity of the crane slew.

Integral functional (12) along with expressions (14), ..., (24) and boundary conditions (25) represent an optimisation problem, in which it was necessary to determine the law of change of the angular coordinate of the payload slew $\psi(t)$ that minimises the value of criterion (12) and meets constraints (14), (15), as well as boundary conditions (26).

The presented variational problem was nonlinear, therefore an approximate method to solve it was used. The unknown function $\psi(t)$ was presented as polynomial with two terms to solve the variational problem:

$$\psi(t)=\psi_1(t)+\psi_2(t). \tag{27}$$

In dependency (27), the first term $\psi_1(t)$ was the selected as a polynomial that meets boundary conditions (26), and the second one – $\psi_2(t)$ – was a polynomial that includes free unknown coefficients, and the following zero boundary conditions are met:

$$\begin{gathered} \psi_2(0)=\dot{\psi}_2(0)=\ddot{\psi}_2(0)=\dddot{\psi}_2(0)=\psi_2^{IV}(0)=\psi_2^{V}(0)=0; \\ \psi_2(t_1)=\dot{\psi}_2(t_1)=\ddot{\psi}_2(t_1)=\dddot{\psi}_2(t_1)=\psi_2^{IV}(t_1)=\psi_2^{V}(t_1)=0. \end{gathered} \tag{28}$$

The function $\psi_1(t)$ was preцsented as the eleventh order polynomial that allows for meeting boundary conditions (26):

$$\psi_1(t)=\sum_{k=0}^{11} C_k t^k. \tag{29}$$

Since in boundary conditions (26) the derivatives with respect to time of the function $\psi(t)$ were included up to and including the fifth order, the higher derivatives were determined:

$$\begin{gathered} \dot{\psi}_1(t)=C_1+2C_2t+3C_3t^2+4C_4t^3+5C_5t^4+6C_6t^5+ \\ +7C_7t^6+8C_8t^7+9C_9t^8+10C_{10}t^9+11C_{11}t^{10}; \\ \ddot{\psi}_1(t)=2C_2+6C_3t+12C_4t^2+20C_5t^3+30C_6t^4+ \\ +42C_7t^5+56C_8t^6+72C_9t^7+90C_{10}t^8+110C_{11}t^9; \\ \dddot{\psi}_1(t)=6C_3+24C_4t+60C_5t^2+120C_6t^3+210C_7t^4+ \\ +336C_8t^5+504C_9t^6+720C_{10}t^7+990C_{11}t^8; \\ \psi_1^{IV}(t)=24C_4+120C_5t+360C_6t^2+840C_7t^3+ \\ +1680C_8t^4+3024C_9t^5+5040C_{10}t^6+7920C_{11}t^7; \\ \psi_1^{V}(t)=120C_5+720C_6t+2520C_7t^2+6720C_8t^3+ \\ +15120C_9t^4+30240C_{10}t^5+55440C_{11}t^6, \end{gathered} \tag{30}$$

where, C_0, C_1, ... C_{11} – the constants that were determined based on motion boundary conditions (26). As a result of the substitution of boundary conditions (26) into expressions (29) and (30), the following values might be obtained:

$$C_0=C_1=C_2=C_3=C_4=C_5=0. \tag{31}$$

The constants C_6, C_7, ..., C_{11} were determined based on the following system of linear algebraic equations:

$$\begin{gathered} C_6+C_7t_1+C_8t_1^2+C_9t_1^3+C_{10}t_1^4+C_{11}t_1^5=\frac{\omega}{2t_1^5}; \\ 6C_6+7C_7t_1+8C_8t_1^2+9C_9t_1^3+10C_{10}t_1^4+11C_{11}t_1^5=\frac{\omega}{t_1^5}; \\ 30C_6+42C_7t_1+56C_8t_1^2+72C_9t_1^3+90C_{10}t_1^4+110C_{11}t_1^5=0; \\ 120C_6+210C_7t_1+336C_8t_1^2+504C_9t_1^3+720C_{10}t_1^4+990C_{11}t_1^5=0; \\ 360C_6+840C_7t_1+1680C_8t_1^2+3024C_9t_1^3+ \\ +5040C_{10}t_1^4+7920C_{11}t_1^5=0; \\ 720C_6+2520C_7t_1+6720C_8t_1^2+15120C_9t_1^3+ \\ +30240C_{10}t_1^4+55440C_{11}t_1^5=0. \end{gathered} \tag{32}$$

As a result of the substitution of constants (31) as well as the ones determined when solving the system of linear algebraic equations (32) into dependency (29), the function that meets boundary conditions with respect to the coordinate ψ and its derivatives with respect to time were determined.

The $\psi_2(t)$ polynomial is represented as follows:

$$\begin{gathered} \psi_2(t)=\left(\frac{t}{t_1}\right)^6\left(\frac{t_1-t}{t_1}\right)^6\cdot \\ \cdot\sum_{k=0}^{n} D_k\left(\frac{t}{t_1}\right)^k\frac{\omega t_1}{2},\ 0\le t\le t_1, \end{gathered} \tag{33}$$

where $D_0, D_1, \ldots, D_n$ – the free coefficients, on which the value of optimisation criterion (12) depends; the multiplier that ensures compliance with zero boundary conditions (27) at arbitrary values of the coefficients D_0, D_1, ..., D_n. These coefficients are free and they ensure the determination of the minimum value of criterion (12).

As a result of the substitution of expressions (29), ..., (33), considering boundary conditions (28), into dependency (27) the expression of function $\psi(t)$ which includes the unknown free coefficients $D_0, D_1, \ldots, D_n$ was obtained.

Using dependencies (17) and (21) the expressions of the angular coordinates of the drive mechanism slew and the rotation part of the crane according to i that also depend on the free coefficients D_0, D_1, ..., D_n were found. At integration, criterion (12) also became the function of the free coefficients $D_0, D_1, \ldots, D_n$. Thus, the approximate solution of the optimisation problem (12), ..., (15) was reduced to determining the optimisation criterion minimum as a function of many variables $D_0, D_1, \ldots, D_n$.

The metaheuristic *VCP-PSO* method (Romasevych *et al.*, 2022) was applied to solve the formulated optimisation problem. At that, the generalised criterion of optimisation (25), considering the expression (12), as the function that depended on 6 free coefficients was represented:

$$C_r=C_r(D_0, D_1, \ldots, D_5). \tag{34}$$

The calculations of the optimal force acceleration mode of the tower crane rotation mechanism at steady payload hoisting based on the criterion of the RMS value of the drive torque (12) and constraints on the drive torque (14) and the power (15) of the drive when meeting the boundary conditions (26) were carried out at such values of the parameters of the jib system: $m = 5000$ kg, $m_2 = 300$ kg, $J_1 = 5{,}51 \cdot 10^5$ kg·m², $J_2 = 4{,}92 \cdot 10^6$ kg·m², $M_0 = 39890$ Nm, $C = 6{,}63 \cdot 10^6$ Nm/rad, $a = 20$ m, $v = 0{,}5$ m/s, $\omega = 0{,}075$ rad/s, $l_0 = 10$ m, $t_1 = 5{,}0$ s, $g = 9{,}81$ m/s², $M_{min} = 0$, $M_{max} = 260000$ Nm, $P_{min} = 0$, $P_{max} = 14000$ W.

RESULTS AND DISCUSSION

The following values of the free coefficients were obtained as a result of solving the optimisation problem:

$$D_0 = 61.88;\ D_1 = -47.38;\ D_2 = 3.58;$$
$$D3 = -9.66;\ D_4 = 1.52;\ D_5 = 90.33.$$

Based on the obtained solution of the optimisation problem of the acceleration process of the tower crane rotation mechanism at steady payload hoisting, graphic dependencies of kinematic (Fig. 2-4), dynamic, and energy characteristics of the jib system of the tower crane in the process the rotation mechanism acceleration were built.

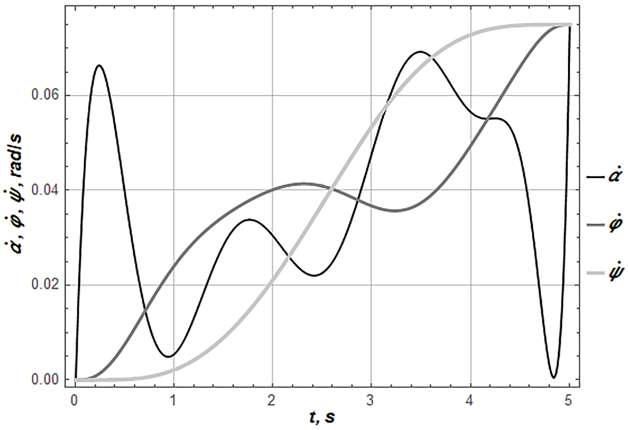

Figure 2. Plots of angular velocities of the rotation mechanism drive , rotation part of the crane and slew of the payload
Source: compiled by the authors

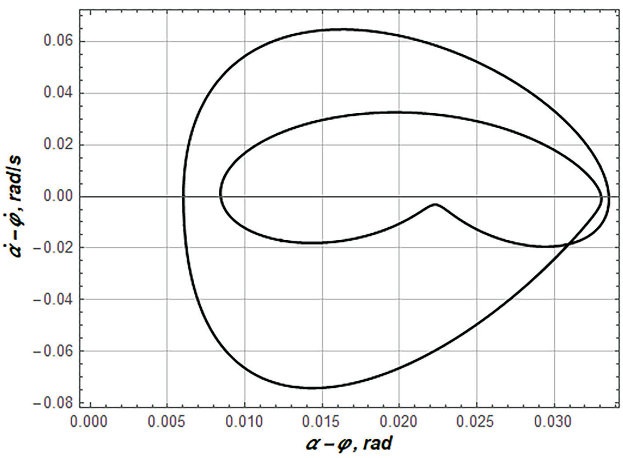

Figure 2. Phase portrait of the elastic deformation of the crane's mast
Source: compiled by the authors

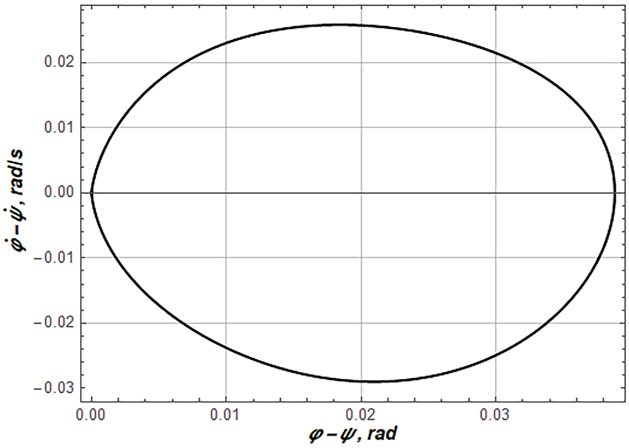

Figure 4. Phase portrait of the cable deviation from the vertical in the plane of the slew
Source: compiled by the authors

From graphic dependencies of the angular velocities of the elements of the jib system of the crane (Fig. 2) conclusion can be drawn that the velocity of the payload slew changes smoothly according to an S-shaped diagram. The velocity of the rotation part of the crane also exhibits a pattern similar to a smooth change. Along with that, the velocity of the drive rotation mechanism exhibits an oscillatory pattern with abrupt increases and decreases of the motion rate. Such a mode of the crane's motion ensures optimal technological characteristics of the crane's performance. At that, the reliability of the drive mechanism's performance may be substantially decreased. This was caused by an abrupt change of motion velocity, it led to the appearance of significant dynamic payloads in the elements of the drive.

Phase portrait of oscillations in the rotation mast of the crane (Fig. 3) shows that oscillations attenuate quite rapidly (within the course of two periods of oscillation). Overall, such motion mode ensures a stable mode of operation of the crane structure. From the phase portrait of oscillations of the payload on the elastic suspension (Fig. 4) it can be observed that during the acceleration process, the oscillations of the payload attenuate smoothly within the course of one period of oscillation and completely disappear upon reaching the steady motion of the rotation mechanism. The obtained phase portrait confirmed the smooth motion of the payload during the operation of the crane rotation mechanism.

The drive torque of the crane rotation mechanism drive exhibits an oscillatory pattern, increasing and decreasing twice from the steady to the maximum value. At that, the maximum value of the drive torque corresponds to the maximum permissible value, which is *260 kNm*. Elastic torque in the rotation mast of the crane also exhibits a similar pattern of change in the process of rotation mechanism acceleration (Fig. 5). However, it takes slightly smaller numerical values in comparison to the drive torque.

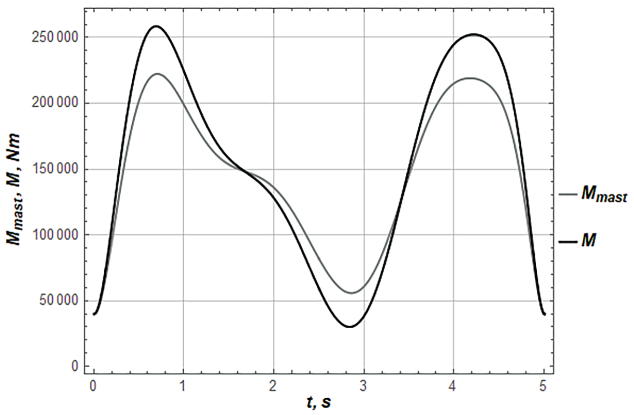

Figure 5. Plots of the drive torque of the crane rotation mechanism drive M and the elastic torque in the rotation mast of the crane
Source: compiled by the authors

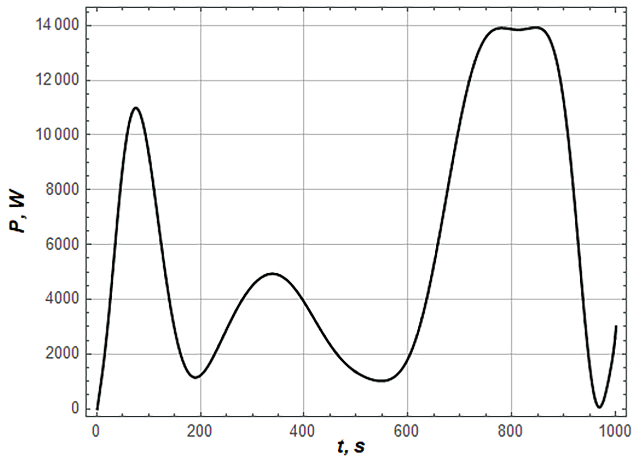

Figure 6. Crane rotation mechanism drive power plot
Source: compiled by the authors

The power of the crane rotation mechanism drive (Fig. 6) exhibits an aperiodic oscillatory pattern with variable frequency and amplitude, reaching the maximum permissible value of *14 kW* in the end of the acceleration process. Upon reaching the steady motion mode of the crane rotation mechanism, the power consumption was *3.0 kW*. The power component of the drive consumed by the deformation of the mast (Fig. 7) initially sharply increases to the maximum value of *9.5 kW* and then decreases to *9.0 kW*. At that, during the acceleration process, all the energy of the crane's mast deformation is returned to the system.

As a result of solving optimisation problem (12), ..., (15), numerical values of kinematic, dynamic, and energy indicators were calculated for the base case (boundary conditions of the rotation mechanism acceleration are ensured) and for the optimal acceleration mode, where additionally a minimum value of generalised criterion (25) is ensured (Table 1).

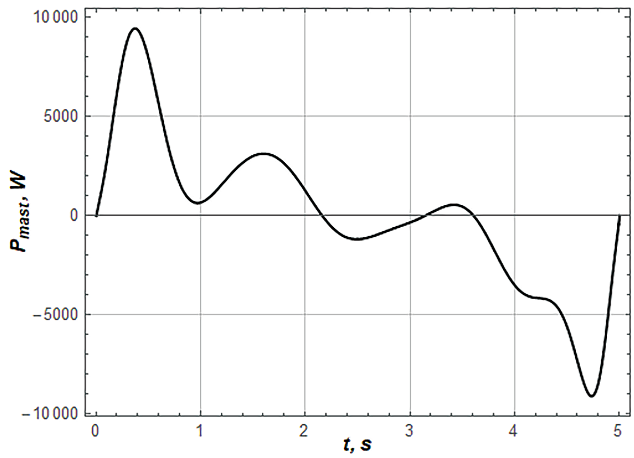

Figure 7. Plot of the change of the component of the drive power that is consumed by the deformation of the rotation mast of the crane
Source: compiled by the authors

Table 1. Numerical values of indicators that correspond to the base and optimal solutions of problem (12), ..., (15)

Estimate evaluation	Unit of measure	Magnitude of the indicator obtained for the solution	
		base	optimal
RMS indicators domain			
Payload deviation	rad	0.0232	0.0223
Elastic deformation of the mast	rad	0.0250	0.0235
Torque in the mast	Nm	165494	155684
Drive torque	Nm	184731	170555
Drive power	W	7892	7378
Maximum indicators domain			
Payload deviation	rad	0.0424	0.0388
Elastic deformation of the mast	rad	0.0386	0.0335
Torque in the mast	Nm	255618	222343
Drive torque	Nm	295608	258573
Drive power	W	17547	13924

Source: compiled by the authors

Analysis of Table 1 showed that compared to the base motion case, the optimal acceleration mode is better in all defined indicators. So, for example, the RMS deviation from the vertical for the elastic payload suspension decreases by

4.0%, and the maximum – by 9.3%. The RMS value of the mast elastic deformation decreases by 6.4%, and the maximum – by 15.2%. The RMS value of the drive torque decreases by 10.8%, and the maximum – by 14.3%, while the RMS value of the drive power decreases by 7.0%, and the maximum – by 26.0%.

In the work by R. Buczkowski & B. Żyliński (2021) the strength and wear characteristics of crane elements were evaluated using numerical analysis. The authors of the article I. Umaru *et al.* (2021) developed a control system that eliminates vibrations and ensures the desired positioning of the payload in gantry cranes. A. Mohammed *et al*. (2023) proposed a method for eliminating residual oscillations of the payload when moving the crane by using a modified controller. In the work by E. Jarzębowska *et al.* (2020) the vibrations of rigid and flexible elements of the crane and the payload with given constraints were analysed. J. Wang *et al.* (2022) proposed a method for eliminating oscillations of the payload and the structure of a floating crane with 3-DOF. In the study by I.A. Martin *et al.* (2021), a controller was developed to monitor oscillations in the payload of a floating crane with a specified number of degrees of freedom (DOF).

The paper by J. Ye & J. Huang (2023) investigated the dynamics of payload transportation by crane reloaders. Here, the main focus was on the influence of oscillations of the payload on a flexible suspension on the dynamics of the structure and the positioning of the payload. The authors M. Bello *et al.* (2024) presented modelling and analysis of the dynamic characteristics of a double-pendulum overhead crane model. The study of the dynamics of motion of an overhead crane with a new wheel design was the subject of the work by N. Fidrovska *et al.* (2021), where stresses in the crane structure elements were determined. In the work by R. Miranda-Colorado (2021) presented a new robust controller to dampen the oscillation of the deflection angle of a crane payload suspension with a variable cable length. Dynamic payloads and swinging of the payload on a flexible suspension also affect its positioning accuracy during loading, unloading, and assembly operations (Kovalenko *et al.*, 2023). In the article by S.M. Fasih *et al.* (2020), control of the payload rotation mechanism was carried out by using a neural network to generate the input data for a jib crane. In the work by S. Tong *et al.* (2024), a method for controlling the dynamic sliding mode has been proposed, and a dynamic model of a double-pendulum cable crane with wind disturbance was developed. The authors of the article by Y. Qian *et al.* (2022) developed a method for controlling the movement of jib crane mechanisms. The work by F. Lui *et al.* (2021) deals with the issues regarding the control of payload swinging and the vibration characteristics of jib cranes, which allowed reducing the payloads on the structural elements. Achievements of optimisation using swarm intelligence methods were presented in the works of another studies by E.H. Houssein *et al.* (2021), T.M. Shami *et al. (*2022) and M. Isiet & M. Gadala (2020).

In work by S.M. Fasih *et al.* (2020), the dynamics of the rotation mechanism of the jib crane were modelled and the deviation from the vertical of the flexible suspension of the payload was determined. To reduce the payload oscillations, the length of the flexible suspension with the payload was changed. Its maximum deviation from the vertical was 10 degrees without changing the length of the flexible payload suspension. The authors managed to reduce its maximum deviation from the vertical to 6 degrees, and with an additional reduction of its oscillations – to 3 degrees by varying the length of the flexible suspension of the payload. The optimisation of the process of starting the rotation mechanism during steady hoisting of the payload made it possible to reduce the maximum deviation from the vertical of the flexible suspension to 2.3 degrees. The authors of the article F. Lui *et al.* (2021) conducted a study of the dynamics of the movement of a jib crane, where the maximum deviation from the flexible suspension of the payload at a steady mode of movement was determined, which is 0.01 radian, and the oscillations of the payload practically do not subside until the end of the movement cycle. As a result of optimising the starting mode of the rotation mechanism when entering the steady mode of movement, the oscillations are completely eliminated. In work by S. Chwastek (2020), the parameters of the jib hoisting mechanism were optimised, where integral quadratic functionals were used as optimisation criteria. In the article, the optimisation of the movement mode of the rotation mechanism was carried out, where also the integral quadratic functional was used as the optimisation criterion, which is the RMS value of the driving torque. This indicates the relevance and expediency of using integral quadratic functionals as criteria in crane mechanism optimisation problems. The article's authors Q. Wu *et al.* (2020) carried out a dynamic analysis and optimisation of the motion mode of the trolley of the bridge crane with a two-pendulum payload suspension. The dynamic analysis showed that practically undamped oscillations of the payload on the flexible suspension are observed. By choosing the start-up duration, it was possible to eliminate the oscillations of the payload on the flexible suspension during the steady motion, but at the same time, in the start-up mode, the maximum deviation from the vertical of the flexible suspension of the payload increased by 1.14 times. In the article, as a result of the optimisation of the start-up mode, the oscillations of the payload on the flexible suspension in the section of steady motion are also eliminated. At the same time, the duration of the start-up process and the maximum value of the deviation from the vertical of the flexible payload suspension do not increase. In the work I. Umaru *et al.* (2021), a dynamic analysis of the movement of a trolley with a payload was performed without and with the use of a PID controller. The analysis of the obtained research results showed that without the use of the controller, the oscillations of the payload are practically not dampened and the maximum value of the deviation from the vertical of the flexible suspension of the payload is 0.11 radians. When using the controller, the maximum value of the deviation from the vertical of the flexible suspension of the payload decreased to 0.06 radians in the start mode, and in the steady mode, these oscillations are practically eliminated. In the article, the oscillations of the payload on the flexible suspension after the start process are also eliminated, but the maximum value of the deviation from the vertical of the flexible payload suspension in the start area is smaller and is 0.04 radians.

The results obtained in the article and in the articles of other authors are close in terms of content and numerical characteristics, and in some cases are even slightly better in similar studies.

CONCLUSIONS

The article provides the results of optimising the motion mode of the rotation mechanism of the tower crane with a beam jib at steady payload hoisting. The tower crane rotation mechanism at steady payload hoisting was presented as the dynamic model with 4DOF, where the main motion of the drives and oscillations of the crane mast and payload on elastic suspension in the plane of the slew were considered. At that, the payload hoisting coordinate was known since the hoist was performed at a constant velocity.

For such a model a system of differential equations of motion of the rotation mechanism at steady payload hoisting is formulated which was reduced to the angular coordinate of the payload slew and its derivatives with respect to time up to and including the sixth order. To decrease dynamic payloads in the elements of the crane and drive structure, an optimisation problem was stated and solved. In this problem the acceleration mode of the rotation mechanism at minimisation of the integral criterion and ensuring the necessary boundary conditions for the jib system motion and constraints on force and energy characteristics was determined. The optimisation criterion used was the RMS value of the drive torque of the rotation mechanism drive during the acceleration process. The constraints included minimum and maximum values of the drive torque and drive power. The integral criterion along with the constraints on the drive torque and drive power was presented as a generalised criterion which was minimised as it reflects undesirable properties of the system.

The presented optimisation problem for selecting the acceleration mode of the rotation mechanism at steady payload hoisting is nonlinear, so an approximate method was applied for its solution. In this problem, the unknown function of the payload slew was represented by a sum of two polynomials. The first polynomial was selected from the condition of meeting the boundary conditions of the rotation mechanism acceleration, and the second polynomial included free coefficients and ensures zero boundary conditions. The values of the free coefficients were determined based on the condition of the minimisation of the generalised criterion using the modified metaheuristic *VCP-PSO* method.

Based on the performed optimisation, an acceleration mode of the crane rotation mechanism which minimises dynamic payloads, reduces the drive mechanism power, and eliminates oscillations of the elements and payload in the steady motion region was obtained. The analysis of the base and optimal acceleration modes of the rotation mechanism shows that in the latter motion mode compared to the former the maximum values of the elastic payload suspension deviation from the vertical decrease by 9.3%, mast deformations by 15.2%, drive torque by 14.3%, and drive power by 26.0%.

Prospects for further research are aimed at optimising the operation of tower cranes through the improvement of dynamic models of their mechanisms. It is proposed to pay special attention to the development of intelligent control systems that allow the implementation of the determined optimal modes of motion of the tower crane rotation mechanism in practice.

An important direction is to increase the energy efficiency of cranes by optimising energy consumption by mechanism drives. The development of hybrid control systems that combine classical PID controllers with fuzzy logic algorithms is promising. This will allow more effective adaptation of systems to changing operating conditions.

Further research involves experimental verification of models in real conditions to confirm their adequacy. This will help optimise the design, reduce wear of elements and increase the safety of crane operation. Modelling taking into account a larger number of factors that will allow more accurate prediction of the behavior and reliability of the crane.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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Оптимізація силового режиму пуску механізму повороту баштового крану при усталеному підйомі вантажу

Анотація. Метою роботи було оптимізувати силовий режим пуску механізму повороту при усталеному підйомі вантажу баштового крана з балочною стрілою. Механізм повороту крана представлено динамічною моделлю з чотирма ступенями вільності, де враховано основний рух механізмів повороту та підйому вантажу, а також коливання поворотної частини крана та вантажу в площині повороту. Для такої динамічної моделі складено математичну модель, яка представляла собою систему звичайних диференціальних рівнянь другого порядку. Систему рівнянь зведено до кутової координати повороту вантажу і представлено рівнянням шостого порядку. Оптимізацію режиму пуску механізму повороту при усталеному підйомі вантажу здійснено за критерієм середньоквадратичного значення рушійного моменту приводу. При цьому враховано обмеження на рушійний момент та потужність приводу. Критерій оптимізації разом з обмеженнями зведено до узагальненого критерію. При оптимізації за крайові умови руху взяті переміщення та швидкості узагальнених координат механізму повороту на початку та в кінці процесу пуску. За результатами оптимізації отримано режим пуску механізму повороту крана при усталеному підйомі вантажу. Отриманий режим мінімізує дію динамічних навантажень, зменшує максимальні значення деформації колони крана, потужності приводу та відхилення вантажного канату від вертикалі. При цьому усуваються коливання ланок крана і вантажу при виході на усталений режим руху. Результати дослідження можна використати у системах автоматизації для керування механізмами повороту баштових кранів, що забезпечує покращену ефективність і стабільність роботи при підйомі вантажу

Ключові слова: стріловий кран; механізми підйому та повороту; динамічні навантаження; оптимальне керування; коливання вантажу