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Influence of regenerative effect and mode coupling on the stability of milling operations

Abstract. Quality of milling operations requires dynamic stability of the process, which depends on the tool geometry, structural rigidity and type of vibrations. The study aimed to identify the influence of internal dynamic disturbances on the milling process, in particular, the mechanisms of self-excited vibrations arising from vibrational interactions and residual irregularities of the machined surface. The methodology included a comparative analysis of milling stabilisation models with an assessment of their advantages, limitations and practical recommendations for implementation in real production processes. Mathematical models were presented that addressed the effects of time delay and the interaction between the directions of oscillation of the tool and the workpiece. The study determined that repeated contact of the tool with the irregularities of the pre-treated surface leads to a variable thickness of the material layer and forms an unstable operating mode. The interaction between different oscillatory modes, when machining flexible or thin-walled parts, leads to more complex self-oscillatory processes, including chaotic oscillations. The most promising for practical use are models with adaptive control, which can respond to changes in process parameters in real time. The study established that the dynamics of the milling process significantly depend on the geometry of the tool, the length of the protrusion, stiffness, mass-inertial characteristics, and wear of the cutting part. Process damping, caused by the side contact of the tool with the workpiece, reduces the amplitude of vibrations in the low-frequency range. The control structure based on the system's state vector included position, speed, vibration amplitude, spindle speed and tool feed. None of the methods is universal, so it is advisable to apply an adaptive approach to choosing a stability model, considering constructive, dynamic and economic factors. The practical value of the results is that they can be used by engineers and technologists to implement adaptive milling control to reduce vibrations and improve machining accuracy

Keywords: self-oscillation; delayed system; multimodal dynamics; semi-sampling; state vector; intelligent control; adaptive modelling

INTRODUCTION

The stability of milling processes is one of the key factors determining the quality, productivity and economic efficiency of 21st-century machine building. Increased attention is being paid to the phenomenon of vibration chatter, a self-excited oscillation that can cause significant deterioration in machining, tool wear and damage to

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the workpiece. The two most influential mechanisms that cause chatter are the regenerative effect and mode coupling. The complexity arises from the simultaneous action of these two types of chatter, as their interaction creates additional difficulties in predicting stability limits. This is driving the growing use of flexible and complex structures in industries such as aerospace, micromechanics, orthopaedic surgery, and automated manufacturing.

The regenerative effect and mode coupling significantly complicate the prediction of milling process stability in conditions of variable stiffness or high-speed machining, and the problem of double chatter (regenerative and mode chatter simultaneously) is a significant aspect. Neglecting their interaction can lead to inaccurate stability predictions, as confirmed by M. Mobaraki & J. Ratava (2023). The study found that only adaptive vibration absorption systems tuned to the frequency overlap of the modal and regenerative chatter can provide effective vibration damping and, accordingly, significantly expand the limits of stable operation. The study developed an adaptive tuning strategy for the Tunable Vibration Absorber (TVA), which extended the limits of stable processing and, for the first time, addressed the frequency overlap between the two types of chatter.

A significant area of research was the influence of tool geometry on the appearance of self-oscillations. For example, machining parts of complex shapes with deep cavities is accompanied by specific instabilities, as proved in the study by X. Wang *et al.* (2024). The study confirmed that the use of curved blades leads to a significant narrowing of the stability region and, based on this, developed refined recommendations for the selection of geometric parameters of cutters for such conditions, which improved forecasting methods and rendered the results more suitable for practical applications. Generalisation of approaches to chatter suppression, including a combination of active and passive vibration control methods, determined their effectiveness in different production conditions. Z. Li *et al.* (2024) found that hybrid damping technologies demonstrate maximum efficiency in a wide range of production conditions. This provides the ability to flexibly configure self-oscillation suppression systems depending on the specific situation and justifies the optimality of the combined approach for a wide range of machining conditions.

C. Liu *et al.* (2024) proposed a new numerical model capable of taking into account the characteristics of various types of parts, from thin-walled to massive, and obtained results that indicate a significant increase in the accuracy of predicting critical cutting modes without the need for large experimental databases and made it possible to use such models in real production conditions without significant costs for experimental verification. Configuration-dependent dynamics of robotic milling systems is also a promising area of research. B. Li *et al.* (2025a) demonstrated that the change in stiffness and modal parameters along the manipulator's trajectory can be used for accurate prediction of the appearance of unstable areas and thus significantly increases the efficiency of automated production. The created model predicts unstable areas with high accuracy, which provides a new level of stability for robotic milling of complex parts.

The need to accommodate the spatial variation of the modal parameters of milling systems was presented in the study by H. Tian *et al.* (2024), dedicated to the stability of machining in orthopaedic robotic systems. The study determined that even minor changes in the tool position cause fluctuations in modal frequencies, which directly affect the probability of chatter. The study quantified this dependence and built more accurate stability models for position-sensitive objects, which is key for milling complex geometric surfaces.

The Ukrainian scientific community is also increasingly emphasising the problem of self-oscillations in milling operations, particularly in the context of machining thin-walled and flexible parts. The problem of self-oscillations in milling operations is critical to the accuracy and quality of machining. S. Dyadya *et al.* (2022) investigated the relationship between vibration parameters and the quality of the moulded surface during milling. The study found that even low-frequency vibrations can significantly worsen the microgeometry of the machined surface, especially in thin-walled or flexible workpieces. This laid the groundwork for practical strategies to improve process stability. To ensure safe and efficient machining of thin-walled parts, it is essential to determine the zones of stable operation without chatter. Y.V. Petrakov *et al.* (2024) addressed the adaptive determination of cutting parameters that prevent the occurrence of self-oscillations. The study significantly contributed to the methodology for calculating chatter-free modes, adapted to the conditions of variable stiffness of structures. Of value is the integration of this methodology into computer numerical control systems, which made it possible to provide real vibration control during machining in an automated environment. The importance of integrating advanced digital and additive technologies in machining is also highlighted in research V. Doroshenko & O. Yanchenko (2023) emphasised the prerequisites for the introduction of 3D technologies in the production of metal products, demonstrating that process innovations not only expand manufacturing capabilities but also set new requirements for stability and quality assurance in machining operations. This corresponds with the growing trend towards adaptive and hybrid approaches in vibration suppression strategies.

Despite previous studies, most existing models are limited in their applicability in conditions of variable real-time dynamics. The influence of combinations of regenerative and mode chatter in adaptive milling processes has not been sufficiently investigated. Problems also arise in the optimisation of robotic systems, where traditional methods do not address variable configuration and multiphase dynamics. Linear models do not include position-dependent dynamics, multimodality, and nonlinear phenomena, which are critical for real-world production conditions.

Therefore, the study aimed to determine the regularities of interaction of internal sources of self-oscillations in high-speed machining processes and to generalise effective approaches to ensuring stable dynamics of milling equipment under conditions of complex vibration interaction. To achieve this goal, the following tasks were set: to classify the main types of self-oscillations that occur during milling, taking into account the features of the geometry, stiffness and tool trajectory, to analyse the effectiveness of mathematical and numerical models for predicting stability in complex machining dynamics, and to formulate practical recommendations for choosing the appropriate stability model depending on the type of task, design constraints and type of equipment.

MATERIALS AND METHODS

The study analysed the dynamic effects that affected the stability of the milling process, in particular, oscillations resulting from feedback and mode interaction. The criteria considered included the accuracy of predicting stability limits, the influence of geometric and dynamic tool characteristics, adaptability to changing machining conditions, and the possibility of practical implementation in high-precision manufacturing.

The study included 3 stages. At the first stage, a theoretical analysis of the mechanisms of dynamic instability of the milling process was conducted regenerative chatter and mode coupling. A mathematical model based on time-delay differential equations was presented, which considered the repeated contact of the cutter with the machined surface and the interaction between the vibration modes. The stage aimed to form an analytical basis for identifying the conditions of self-oscillations and determining the limits of stability depending on technological and design parameters. The presented model became the basis for the construction of the Stability Lobe Diagram (SLD) and justification of further approaches to process stabilisation. Time-delay system, where the change in chip thickness depends on the current tool position and the previous tool position by a time (delay) (Insperger *et al.*, 2015):

$$h(t) = x(t) - x(t - T), \quad (1)$$

where: $h(t)$ – Clip thickness at the current moment; $x(t)$ – current tool position; $x(t - T)$ – previous tool position at time T (time delay between the current and previous tool contact with the workpiece due to the regenerative effect). This describes the key characteristic of the model, the dependence of chip thickness on the current and previous tool position. It is this relationship that reflects the regenerative effect, which is one of the main causes of vibrations in milling. Without this equation, it is impossible to describe the effect of lag on the cutting process.

The cutting force *F(t)* is defined as proportional to the chip thickness as follows (Faassen *et al.*, 2003):

$$F(t) = K_h \times h(t) = K_h \times (x(t) - x(t - T)), \quad (2)$$

where: $F(t)$ – force acting on the tool during machining, which depends on the chip thickness, which varies due to tool movement and machining geometry; K_h – system stiffness factor (may depend on the workpiece material, tool type and other physical characteristics of the system). The equation defines the cutting force as a function of chip thickness, which directly links the process mechanics to the geometrical parameters of the cut. This basic relationship quantifies the interaction between the tool and the workpiece and is the starting point for further stability analysis.

The motion of the tool is described by Newton's second law or the equation of a one-degree-of-freedom oscillatory system (Breitenbach, 2025):

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t), \quad (3)$$

where: m – tool mass; c – damping coefficient; k – stiffness coefficient; $\dot{x}(t)$ – tool velocity at time t, i.e., tool displacement rate; $\ddot{x}(t)$ – tool acceleration at time t. This equation summarises the effects of tool mass, stiffness and damping on tool vibrations due to cutting forces. Without this equation, it is impossible to conduct a mathematical analysis of the dynamic stability of the process.

At the second stage of the study, a comparative analysis method was used to systematise existing approaches to modelling and ensuring the stability of the milling process. A comparative typology of models was formed: three-dimensional Stability Lobe Diagram (3D SLD) with positional dependence (analysis of the influence of the tool's spatial position and local stiffness on the stability limit), Nonlinear Delay Differential Equation (NDDE) models with bifurcations (describing the behaviour of systems with self-oscillations, bifurcations and chaotic modes, which is critical for flexible parts), variable spindle speed methods (practical strategies for suppressing chatter by modulating the input signal without modifying the design), 6 Degrees of Freedom (6DOF) robotic system models (accounting for the kinematics of robotic systems with multiple degrees of freedom), models with uncertain parameters (accounting for statistical fluctuations in stiffness and damping typical of manufacturing conditions), and multimodal milling approaches (modelling the interaction of multiple frequencies simultaneously – a typical situation in complex machining paths). The models were selected to cover the most common destabilisation scenarios that can occur when milling different types of workpieces, with different geometries, stiffnesses, and in different design conditions (traditional and robotic systems). This concluded on the relevance of using certain methods depending on the production scenario.

At the third stage, the results of the comparative analysis were summarised, which formulated practical recommendations for choosing the most effective methods of milling process stabilisation for different production scenarios. For each of the approaches, the advantages, limitations in practical implementation, and recommendations for use in mass production were analysed separately.

Evaluation criteria included computational complexity, sensitivity to input parameters, flexibility of adaptation to different materials and types of workpieces, and compliance with industry requirements (aerospace, mechanical engineering, robotic systems). This typological approach identified the most relevant methods for specific production tasks and formulated reasonable recommendations for their implementation. The Zotero bibliography manager (version 6.0) was used to organise bibliographic sources, make references and systematise scientific publications, which ensured proper citation accuracy and uniformity of references in the process of preparing the paper.

RESULTS

Mathematical model for predicting stability in milling processes

One of the main problems in milling is dynamic instability, in particular chatter, which reduces machining quality, tool life and accuracy, and is caused by the regenerative effect and mode coupling. Regenerative chatter occurs when the cutter repeatedly contacts a surface that already contains irregularities formed by the previous tooth. This creates a delay in the system, which is a source of self-oscillation. A typical mathematical model is a time-delay system in which the current chip thickness is defined as a function of the previous coordinates. The model is classified as a Differential Delay Equation (DDE) and is used to construct stability limits in the form of SLDs. The use of modified semi-sampling methods incorporates the dependence of stability on tool position and changes in damping.

Regenerative chatter is a typical self-oscillatory process caused by the cutter coming into periodic contact with a surface that has already been machined by a previous tooth. This creates a variable chip thickness and a time delay equivalent to the tool rotation period. This dynamic creates a feedback loop in the system, making it potentially unstable. This effect is modelled as a DDE equation where the chip thickness depends on the previous tool position (Formula 1) and the cutting force is proportional to this thickness (Formula 2). The resulting equation, which includes inertial, damping and elastic components (Formula 3), was used as a basic model for interpreting self-oscillations and is the basis for building an SLD to analyse the stability limits of the milling process.

The phenomenon of mode coupling occurs when two or more interdependent vibration modes are present, particularly in flexible or thin-walled workpieces as well as in multi-axis robotic systems. Typical examples are the interaction of bending and torsion in long tools or in parts with complex geometries. In such conditions, multi-frequency dynamics are observed, which can lead to a loss of stability. To describe this phenomenon, a model with two coupled degrees of freedom is used, where each direction has unique parameters of mass, stiffness, and damping (Faassen *et al.*, 2003):

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = K_h \times (x(t) - x(t - T)). \tag{4}$$

A typical mathematical model for two coupled degrees of freedom – a system of two equations for two different vibration modes (e.g., in the X and Y directions) that covers inertia, damping, and stiffness for each mode – is as follows:

$$\begin{pmatrix} m_x\ddot{x}+c_x\dot{x}+k_x x=F_x(h_x(t)) \\ m_y\ddot{y}+c_y\dot{y}+k_y y=F_y(h_y(t)) \end{pmatrix}, \tag{5}$$

where: m_x, m_y – tool masses in the X and Y directions; $\ddot{x}, \ddot{y}$ – accelerations in the X and Y directions; c_x, c_y – damping coefficients for each direction; k_x, k_y – stiffness in the X and Y directions; $F_x(h_x(t))$, $F_y(h_y(t))$ – forces acting on the tool, which depend on the chip thickness and vary over time due to the regenerative effect. The interaction between the axes is considered through off-diagonal elements in the stiffness and damping matrices, which can be used to model the energy exchange between modes.

For mode coupling, it is critical to consider the interaction between two vibrational modes. This means that a displacement in one direction (e.g., x) can generate a force in the other direction (y) and vice versa. This interaction can be modelled through off-diagonal elements in the stiffness matrix. An extended system to account for the interaction between modes is given by T. Insperger *et al.* (2015):

$$\begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix}[\ddot{x}(t)\ddot{y}(t)] + \begin{bmatrix} c_x & c_{xy} \\ c_{yx} & c_y \end{bmatrix}[\dot{x}(t)\dot{y}(t)] + \\ + \begin{bmatrix} k_x & k_{xy} \\ k_{yx} & k_y \end{bmatrix}[x(t)y(t)] = \begin{bmatrix} F_x(h_x(t)) \\ F_y(h_y(t)) \end{bmatrix}, \tag{6}$$

where: c_{xy}, c_{yx} – damping coefficients that address the interaction between the X and Y directions (a change in velocity in one direction can affect the damping in the other); k_{xy}, k_{yx} – elements of the stiffness matrix that describe the interaction between the X and Y directions (if these elements are non-zero, then a change in one direction (x) causes a change in the other (y)).

For ease of calculation and analysis, the system of equations can be written in matrix form, where each element of the matrix will be responsible for the corresponding value of the system (Insperger *et al.*, 2015):

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t), \tag{7}$$

where: M – mass matrix; C – damping matrix; K – stiffness matrix; $\ddot{u}(t)$ – displacement acceleration; $\dot{u}(t)$ – displacement velocity.

SLD is a key tool for visualising the boundaries between stable and unstable milling operations. It can be used to graphically determine the critical cutting modes at which the system enters an unstable state, accompanied by the development of self-oscillations (chatter). In the classical case, SLD construction is based on solving a regenerative DDE model, with the determination of the delay time based on the rotation parameters. In this case, stability is analysed using a spectrum of Lyapunov's indices, where the transition to instability occurs when one of them is zero. This can be used to calculate the critical plunging depth for each spindle speed. The SLD is constructed by determining the set of pairs (N, a) for which the system enters the unstable mode. This identified the "frontal structures" characteristic of milling, which are shaped as petals on the graph. In the case of two vibration modes (in the X and Y directions), the stiffness matrix is as follows (Faassen *et al.*, 2003):

$$K = \begin{bmatrix} k_x & k_{xy} \\ k_{yx} & k_x \end{bmatrix}. \tag{8}$$

The presence of non-zero off-diagonal elements (k_{xy}, k_{yx}) indicates that displacement in one direction generates forces in the other. This leads to complex self-oscillatory behaviour, such as bifurcations, quasi-regular oscillations or chaotic fluctuations, especially when the local stiffness is low or the damping is poor. In industrial applications, this can occur when machining aluminium or titanium panels, shell structures, or when the tool is unstable or insufficiently rigid. Classic examples include milling long, flexible parts or using six-degree-of-freedom manipulators where geometric complexity and poor damping contribute to resonant excitation. Particularly critical is the incorrect distribution of damping, where compensation of one mode can cause resonance in another. This approach can be used to perform modal analysis – to identify the natural frequencies and vibration forms of the system, as well as to assess the limits of its dynamic stability. Consideration of mode coupling is essential for accurate SLD prediction in milling operations involving complex trajectories or the interaction of several vibrational modes.

In the classical case, SLD construction is based on the Dynamic Data Exchange (DDE) analysis, which describes a regenerative chatter, where the latency is defined as (Faassen *et al.*, 2003):

$$T = \frac{60}{N \times z}, \tag{9}$$

where: N – spindle speed per minute (rpm), i.e., tool rotation frequency (a critical parameter because the rotation speed affects the system's vibration frequency and how fast the tool moves); z – number of teeth on the tool (determines how often the tool comes into contact with the material during machining, which affects the amount of force transmitted through the system); 60 – constant factor used to convert minutes to seconds, ensuring correct calculation of the time for one tooth to pass.

Numerical methods such as the Semi-Discrete Method (SDM), modal analysis, time-domain simulation, and the Galerkin method are used to accurately calculate stability limits (Altintaş & Budak, 1995; Faassen *et al.*, 2003). These methods can build a stability diagram in more complex systems, incorporating the various factors that affect the stability of the machining process. In the stable zone, the solution does not grow over time, and the limit of instability is determined when one of the indicators becomes zero. This calculated the critical cutting depth a_{lim} for each mode (Zheng *et al.*, 2014):

$$a_{lim} = \frac{2 \times K}{h} \times R[G(j\omega)]^{-1}, \tag{10}$$

where: h – chip thickness removed during one tool pass (varies depending on the depth of cut and machining speed); $j\omega$ – complex frequency, where j – conditional unit, and ω – real frequency; $R[G(j\omega)]$ – modulus of the system frequency response function that describes the amplitude of the system frequency response at a certain frequency point $j\omega$.

The dynamics of the milling process depend heavily on the tool geometry, including the length of the protrusion, stiffness, mass-inertial characteristics and wear of the cutting part. Long and thin tools show increased sensitivity to the excitation of bending and torsional modes, which contributes to the appearance of both mode and regenerative chatter. The behaviour of the system is multidimensional with the presence of inter-axial (cross-mode) interaction, which is confirmed by models in the three-dimensional coordinate space (X, Y, Z). SLD is extended to three-dimensional or parametrically dependent diagrams as follows (Altintaş & Budak, 1995):

$$SLD = f(N, a, p), \tag{11}$$

where: a – depth of cut (the distance to which the tool penetrates the material during machining, the depth of cut affects the cutting forces and, accordingly, the stability of the process); p – parameter that determines additional system characteristics (variable stiffness, spindle speed, number of degrees of freedom of the system).

The interaction of oscillations in different directions can lead to multi-frequency modes and loss of stability. The lag caused by the regenerative effect creates conditions for the growth of self-oscillations even in the absence of external disturbances. In a discrete form, this is expressed through the dependence of the cutting force on the previous coordinate values, which reflects a time-delayed differential equation (DDE model). The dynamics of the milling process is largely determined by the tool geometry, i.e., the length of the protrusion, stiffness, mass-inertial characteristics, and the state of the cutting edge (including wear). For a generalised description of an elastic mechanical system with a regulatory delay that incorporates vibration effects and possible multi-frequency modes, the equation of motion is given by T. Insperger *et al.* (2015):

$$M_{\ddot{q}}(t) + C_{\dot{q}}(t) + K_{q}(t) = F_{c}(h(t)). \tag{12}$$

Process damping, caused by the lateral contact of the tool with the workpiece, can reduce the amplitude of vibrations in the low-frequency range. This effect shows a dependence on the cutting speed and depth of cut, addressing the influence of geometry and feed. The actual milling process is three-dimensional, as cutting forces are generated in all spatial directions. Therefore, it is necessary to address the complex dynamic behaviour of the system in

X, Y, Z. System matrices are used to describe the behaviour (Faassen *et al.*, 2003):

$$M_{\ddot{q}}(t) + C_{\dot{q}}(t) + K_q(t) = F_c(q(t), q(t - T)). \quad (13)$$

This representation can address cross-mode interaction (for example, when fluctuations in *X* stimulate fluctuations in *Y*). The delay caused by the regenerative effect creates conditions for the dynamic growth of fluctuations even without an external perturbation.

Due to the complex and multifactorial dynamics of the milling process, classical control methods (linear controllers or fixed models) are ineffective in the face of changing machining modes and structural instability. This substantiates the feasibility of introducing intelligent stabilisation systems based on Deep Reinforcement Learning (DRL). DRL can be used for adaptive control that considers variable system state parameters, such as position, speed, vibration amplitude, spindle speed, and tool feed. In a discrete formulation, it functions as a force dependence on previous coordinate values (Altintasz & Budak, 1995):

$$F(t) = K_c \times [x(t) - x(t - T)]. \quad (14)$$

Process damping is caused by the contact between the tool side and the workpiece, which is particularly evident at low feeds or high plunging depths. This phenomenon is nonlinear in nature and contributes to a reduction in the amplitude of vibrations, mainly in the low-frequency range. The mathematical description of such effects belongs to the class of time-delay equations (DDEs), which require the use of specialised solution methods, such as the Semi-Discretisation Method (SDM), Impedance Response Function (IRF) or Pade approximations. Based on the current state of the system, Deep Reinforcement Learning (DRL) generates a control signal that is fed to the piezoelectric actuator to actively suppress vibrations. The process damping is modelled as an additional force proportional to the tool speed and dependent on the depth of cut (Tuysuz & Altintas, 2019):

$$F_{pd}(t) = c_{pd} \times \dot{x}(t) \times f(d_{axial}), \quad (15)$$

where: $f(d_{axial})$ – function that addresses the dependence of damping on the depth of cut. These equations help to accurately describe the dynamics of the milling process, incorporating the various factors that influence machining stability. Due to the complex dynamics of the milling process, classical control methods are mostly ineffective, which increases the need for intelligent stabilisation systems such as DRL, which provides an adaptive response to changes in machining conditions and reduces vibrations. The state vector in such control systems and modelling of vibration processes, in milling and mechatronic tasks, is represented as (Altintaş & Budak, 1995):

$$s(t) = [x, \dot{x}, a_{vib}, N, a_p], \quad (16)$$

where: a_{vib} – vibration amplitude (a significant indicator of process stability and affects the quality of machining); a_p – parameter that characterises the tool feed during machining. To optimally control the milling process and reduce tool vibrations, a control scheme is used that incorporates the current state of the system (Sutton & Barto, 2014):

$$u(t) = \pi_o(s(t)), \quad (17)$$

where: $u(t)$ – control signal that determines the action to be taken based on the current state of the system; $\pi_o(s(t))$ – function that generates a control signal that depends on the state vector *s(t)*. This described the process of generating a control signal as a function of the current state, without using fixed parameters.

Taken together, these equations provide a universal mathematical description of milling that provides simultaneous analysis of process stability, prediction of critical conditions, evaluation of the effects of geometry, material and machining parameters, and development of effective active vibration suppression strategies. This creates a consistent, multi-level model that combines physical process mechanics, analytical stability methods and adaptive control algorithms.

Classification and recommendations for technological efficiency of chatter reduction methods

Ensuring stability during milling is a critical factor in achieving high precision and productivity. The development of stability models addressed a wide range of factors, from regenerative chatter and mode interaction to the adaptive behaviour of robotic systems, which tailors the process to the specifics of the task. Table 1 presents these approaches, demonstrating the relationship between the type of model, the chosen stabilisation technique and its application in industry.

Table 1. Comparison of stability methods for milling with damping and tool parameters

Model type	Stability analysis and influence of parameters on machining	Methods of stability in processing	Application and scope of use
3D Stability Lobe Diagram with position dependence	Change in stability when changing tool position for thin-walled parts, considering process damping	SDM (semi-discretisation) + positional discretisation of the toolpath to account for changes in stiffness and damping	Building a 3D SLD for milling thin-walled components, analysing the impact of process damping on stability limits, and applications in the aerospace industry. Using 3D SLD to machine thin-walled aluminium aircraft panels where positional dependence reduces regenerative chatter. Optimising the milling of curved turbine blade surfaces to reduce vibrations.
NDDE + nonlinear model	Modal and regenerative chatter, bifurcations, effects of tool wear and part flexibility, chatter suppression with TVA	Galerkin method for projection of nonlinear equations, mode summation for multimodal behaviour	Analysis of stability and chaos zones, use of TVA for chatter suppression, and prediction of bifurcations in nonlinear systems. Application of TVA to suppress mode coupling in the milling of flexible titanium plates in aircraft components. Predicting chaotic behaviour in thin shell machining for aerospace structures.

Continued Table 1.

Model type	Stability analysis and influence of parameters on machining	Methods of stability in processing	Application and scope of use
Variable speed, regeneration	Influence of different forms of rotational speed modulation (sinusoidal, triangular) on the stability limits	Time-domain simulation + SDM for building SLDs with variable velocity $\omega(t)$	Comparison of speed modulation strategies for chatter suppression, optimisation of variable speed machining modes. The use of sinusoidal speed modulation to reduce regenerative chatter in the machining of aluminium parts in the aviation industry. Application of triangular modulation for milling steel components in mechanical engineering.
Robotic 6DOF model	Milling stability for industrial robots with 6 degrees of freedom, considering the flexibility of the arm	Fully discrete method for numerical solution of delayed differential equations	SLD analysis for flexible robotic systems, optimisation of machining modes for robots based on their kinematics. Use of the 6DOF model to stabilise milling in orthopaedic surgery by reducing mode coupling. Optimising the handling of large parts in automated production.
Unspecified parameters	The effect of parameter variation (stiffness, damping) on SLD accuracy in real-world production environments	IRF + probabilistic approach to assessing uncertainties	Assessment of SLD reliability, analysis of the impact of uncertainties on machining stability, and application in production. Application of a probabilistic approach to evaluate the regenerative chatter in titanium alloy milling in mass production. Analysis of the impact of stiffness uncertainties on the stability of composite materials processing.
Multi-modal robotic milling	Stability dependence on robot configuration, multimodal milling dynamics	Stability Cloud Simulation to assess instability zones depending on the trajectory	Trajectory assessment of instability zones, optimisation of robot configuration for milling. Using Stability Cloud Simulation to reduce mode coupling when milling complex parts in robotic systems for the aviation industry. Optimising paths for highly loaded robots in large component manufacturing.

Source: compiled by the authors based on X. Liu *et al.* (2024), D.R.V. Naranjo *et al.* (2024), J. Wu *et al.* (2024), H. Tian *et al.* (2024), K. Guo *et al.* (2025), B. Li *et al.* (2025b), Y. Du *et al.* (2025), J. Jia *et al.* (2025), K. Nasiri & H. Moradi (2025)

In a production environment, the efficiency of the milling process depends not only on the basic parameters of the machining mode, but also on the ability of the system to adapt to instabilities caused by vibrations, variable stiffness, geometric design features or material uncertainties. Various stability models are used for this purpose. Table 2 provides a comparative analysis of the models, indicating their strengths, weaknesses, and practical recommendations for implementation. This approach identified the most relevant methods for various production scenarios, from machining thin-walled structures to milling in complex robotic systems with multimodal dynamics.

Table 2. Analysis of the effectiveness of milling stability methods under production constraints

Model type	Advantages	Disadvantages	Recommendation
3D Stability Lobe Diagram with position dependence	The method addresses positional variations in stiffness and damping, providing accurate prediction of stability limits for thin-walled components widely used in the aerospace industry. The 3D SLD provides a detailed stability map that helps optimise machining conditions for parts with variable geometries. The method integrates damping effects, which improves the accuracy of chatter prediction in real-world conditions. Widespread use in the aerospace industry can be used for machining lightweight structures with increased accuracy and minimal vibration.	SDM requires significant computing resources, particularly for complex toolpaths or large parts. The method is optimised for thin-walled parts, which reduces its effectiveness for massive or rigid components. Correct 3D SLD construction requires accurate measurements of dynamic parameters (stiffness, damping), which can be difficult in real-world conditions. Small deviations in the toolpath may require recalculation of the SLD, which complicates the process.	Milling thin-walled parts in the aerospace industry, such as aircraft panels or turbine blades. Combine with experimental tests to refine stiffness and damping parameters before building an SLD. Use of specialised tools (Matrix Laboratory (MATLAB) or ANSYS) to automate SDM and reduce computational costs. Planning the toolpath to minimise positional changes that affect stability.
NDDE + nonlinear model	The method incorporates mode and regenerative chatter, bifurcations, and chaos, which can be used to analyse the behaviour of flexible parts. The use of TVA ensures effective vibration reduction in real time. The model can estimate the effect of wear on stability, which improves the accuracy of predictions for long machining processes. The Galerkin method provides an analysis of multimodal dynamics, which is effective for complex systems with multiple frequencies.	NDDE analysis requires significant computing resources and specialised software. Setting up and integrating vibration absorbers in an industrial environment can be costly and technically challenging. The effectiveness of TVA depends on the precise determination of system parameters, which requires additional measurements. The method is effective for flexible parts, but less appropriate for rigid structures.	Machining of flexible wafers or parts with a high risk of nonlinear chatter, e.g., in the aviation industry. Conducting laboratory experiments to determine optimal TVA parameters before industrial use. Combination with active controllers (soft-actor-critical or fuzzy controllers) to increase efficiency. Use of numerical modelling tools (DDE-BIFTOOL) to analyse nonlinear effects.

Continued Table 2.

Model type	Advantages	Disadvantages	Recommendation
Variable speed, regeneration	Speed modulation (sinusoidal, triangular) effectively reduces regenerative chatter. No complicated changes to the machine or tool design are required, just the speed setting. Different modulation shapes (sinusoidal, triangular) can be used to adapt the model to different materials and machining conditions. Increases productivity without significant investment in equipment	Efficiency depends on the correct choice of modulation form, which requires preliminary testing. The method is less effective for parts with uneven stiffness or complex trajectories. The choice of modulation frequency and amplitude requires experimental data. Less effective for materials with low regenerative chatting	Processing materials with a high risk of regenerative chatter (aluminium alloys). Conducting tests with different modulation shapes (sinusoidal, triangular) to determine optimal parameters. Using SDM to accurately predict the stability limits at variable speeds. Integrate speed modulation into numerical control systems for automatic tuning.
Robotic 6DOF model	The method considers the flexibility of the manipulator and the kinematics of the robot, which ensures accurate stability prediction. It can be used to configure machining modes for robots with 6 degrees of freedom, which is a priority for manufacturing. The fully discrete method provides fast analysis of delayed differential equations. The method adapts to complex robot trajectories.	The method was developed for robotic systems, which reduces its effectiveness for traditional machine tools. Accuracy depends on detailed data on the kinematics and flexibility of the manipulator. Implementation in real-world conditions may require modification of the robot's software. Less effective for machining rigid parts	Milling in automated production with industrial robots. Conducting experiments to determine the flexibility parameters of the manipulator. Use of SLD to build a robot kinematics-based design. Using a numerical modelling tool (RobotStudio) to optimise trajectories.
Unspecified parameters	The probabilistic approach incorporates the spread of parameters (stiffness, damping), which increases accuracy in real-world conditions. It can be used for predicting the probability of a chatter with uncertain parameters. The method is suitable for various types of machining where parameters can vary. The Impulse Response Function simplifies the analysis of the impact of uncertainties.	Requires specialised software and large amounts of data. Accuracy depends on the quality of the scatterplot measurements. Less effective in stable conditions (the method is less useful if the parameters are known precisely). Probabilistic analysis can be resource-intensive	Use in production environments with a high level of uncertainty (when processing new materials). Collecting data on the spread of parameters using sensors and measuring systems. Use IRF to quickly assess the impact of uncertainties on stability. Use of tools for probabilistic modelling (Monte Carlo in MATLAB).
Multi-modal robotic milling	The method incorporates the dependence of stability on robot configuration and multimodal dynamics. Stability Cloud Simulation provides a visual representation of the chatter zones depending on the trajectory. It can be used to adjust parameters to reduce vibrations. The method is suitable for robotic industries with complex trajectories.	The method is designed for robotic systems, which reduces its effectiveness for traditional machine tools. Stability Cloud Simulation requires significant computing resources. Accuracy depends on detailed data on the robot's trajectory and dynamics. May require modification of the robot software	To optimise milling in robotic systems with multimodal dynamics. Use of Stability Cloud Simulation to estimate instability zones in trajectory planning. Conduct tests to refine the modal parameters of the robot. Use trajectory modelling tools (Robot Operating System (ROS) or CoppeliaSim).

Source: compiled by the authors based on X. Liu *et al.* (2024), H. Miao *et al.* (2024), D.R.V. Naranjo *et al.* (2024), H. Tian *et al.* (2024), J. Jia *et al.* (2025), K. Nasiri & H. Moradi (2025), K. Guo *et al.* (2025), B. Li *et al.* (2025b), K.A. Hassan (2025)

The analysis of the methods showed that there is no one-size-fits-all solution suitable for all types of machining. Models based on 3D Stability Lobe Diagram demonstrate significant accuracy for thin-walled structures when fine-tuned but require significant computational resources. Non-linear NDDE models using TVA are effective for flexible parts but are complex to implement and sensitive to parameterisation. Variable speed methods are simple to implement but have limited effectiveness for complex trajectories and rigid materials. Robotic approaches with 6DOF and multimodal simulations are particularly relevant for automated systems but require detailed kinematic models and substantial computational support. Probabilistic approaches incorporate real production fluctuations but require careful collection of statistical data.

Therefore, the choice of model should be based on specific process characteristics, production goals and the acceptable level of implementation complexity, which emphasises the need for an adaptive and engineering-based approach to ensuring the stability of milling operations. A comprehensive approach to method selection, incorporating practical recommendations and the nature of instabilities, will increase the reliability, accuracy and adaptability of the milling process in high-precision production.

DISCUSSION

The results of the study have shown that the stability of the milling process depends on several factors, including changes in stiffness and damping, nonlinear dynamics with the interaction of mode oscillations, and the use of adaptive stabilisation methods under variable machining conditions. To solve these problems, approaches such as SLD models that consider the spatial variability of parameters, TVA, numerical schemes based on NDDE, and Stability Cloud Simulation for multi-axis robotic systems were used. A similar approach was presented in study C. Song *et al.* (2024), based on time-delayed DDE models to build SLDs using the SDM method, which improved the accuracy of prediction of unstable zones during milling of complex components. This fully correlates with the results of the current study, which emphasised the need to incorporate the spatial dependence of stiffness and damping when constructing stability diagrams.

O. Özşahin (2024) proposed an active electromagnetic toolholder integrated with a feedback control system for effective chatter suppression at low tool immersion. The study implemented an electromagnetic system with active control, which provides dynamic stiffness adjustment during milling. Experimental tests have confirmed that this approach can significantly reduce the amplitude of self-oscillations, ensuring stability and high surface quality of the parts. This is consistent with the current study, confirming the effectiveness and feasibility of using adaptive active stabilisation systems for modes with a shallow immersion depth and low system stiffness. This approach complemented the solutions proposed in this study (DRL and TVA) and confirmed the feasibility of engineering adaptation of stabilisation methods to the conditions of a specific production scenario.

Additionally, R.R. Lakshmi *et al.* (2024) optimised milling mode parameters to reduce chatter. The authors used a combination of numerical analysis and experimental validation to identify the impact of parameters such as cutting speed, feed rate and depth of cut on machining stability. The results demonstrated that the correct selection of parameters can significantly reduce the amplitude of vibrations and improve surface quality without the use of additional stabilisation devices. This correlates with the results of the current study, which substantiated the effectiveness of using adaptive control of technological modes as one of the tools to stabilise the milling process.

Furthermore, the study determined that damping is a crucial factor for machining stabilisation, including under conditions of low local stiffness and the presence of mode coupling. This was confirmed by R.I.E. Schönecker *et al.* (2024) in an experimental investigation of the effect of microstructured tool surfaces on their ability to passively dampen. The study determined that the structured surfaces significantly reduced the amplitude of vibrations without the need to change the machining modes or tool geometry, which complemented the results of the study on the critical role of damping. S. Xin *et al*. (2023) used TVA within the NDDE framework to suppress chatter under complex nonlinear dynamics. They found that TVAs effectively expanded the limits of stable processing by reducing vibration amplitude in complex dynamic situations. This correlated the results of an ongoing study on the effectiveness of TVAs for stabilising systems with pronounced nonlinear mode interaction.

The stability of robotic milling systems with 6DOF, including the interaction between the axes and the flexibility of the manipulator, must be considered. The use of Stability Cloud Simulation was used to visualise unstable zones in the motion paths. Similar results were obtained in the study by Y. Zheng *et al.* (2023), confirming the priority of considering the flexibility of mechanical joints to build accurate stability models. The study also determined that real dynamics of the manipulator can be used to reduce the error in predicting self-oscillations and thus improve the quality of milling. The study emphasised the need for detailed experimental studies of the dynamic parameters of the system, which fully correlated with the results of this study on the key role of parametric accuracy of stability models. J. Du *et al.* (2023) applied a combined robust Linear Quadratic Gaussian (LQG) control to effectively suppress chatter, demonstrating the need for adaptive stabilisation methods, which coincided with the approach of this study to the use of control methods such as DRL. The study showed that an adaptive stabilisation system is significantly more effective than traditional fixed controllers, particularly in conditions of variable part stiffness and machining modes. The study also noted that the robust approach reduces the sensitivity of the system to errors in parameter estimation, which was consistent with the probabilistic analysis of parameter uncertainty proposed in the current study.

Y. Sun *et al.* (2023) reviewed the 21st-century approaches to ensuring the stability of milling thin-walled parts that are particularly sensitive to self-oscillations (chatter). The study systematised and classified existing methods according to their principles of operation, mathematical models, level of adaptability and practical application in industry. They emphasised that there is no single universal solution that can completely solve the problem of chatter in all production scenarios. Stabilisation of the process is only possible through a comprehensive combination of analytical theoretical models and carefully conducted experimental studies. The conclusions were fully confirmed by the results of the current study, which showed the need for an engineering-adaptive selection of stabilisation methods following specific production conditions.

The results of the study demonstrated that parametric uncertainties, such as stiffness and damping fluctuations, affect the accuracy of milling stability prediction. Ignoring them reduces the accuracy of SLD and leads to errors. This correlated with the study of M. Hashemitaheri *et al.* (2024), determining that even small variations in stiffness or damping parameters significantly affect the accuracy of SLD construction. They used the Monte Carlo method to assess the impact of these variations on the modelling results. The study determined that small changes in damping lead to a significant shift in the stability limits, which can cause chatter even in modes that were previously considered stable. This confirmed the need to incorporate parametric uncertainties, which was reflected in this study, and justified the use of probabilistic methods for more accurate stability prediction in production conditions.

S. Hayati *et al.* (2023) proposed an approach with variable tool dynamics (a hollow core cutter with an adjustable core) that can be used for adaptive changes in frequency characteristics during the milling process. The study experimentally confirmed that this tool design can significantly reduce self-oscillation, especially when machining long and thin parts where stability is a critical factor. They also demonstrated that adjusting the dynamic characteristics of the tool directly during operation makes it possible to adapt to different milling conditions, which significantly expands the technological capabilities of the machines. This correlated with the results of a study on the impact of tool design on stability and highlighted the need to consider the geometric and design features of the tool in complex production scenarios.

Thus, successful stabilisation requires a comprehensive adaptive approach that considers the specifics of technological conditions and design features of the equipment and also involves the integration of theoretical models with experimental validation. The applied approaches and methods correlate with current trends in the field of milling stabilisation and can be effectively implemented in production conditions, which will not only increase the stability of machining but also improve the overall technological flexibility and reliability of milling production.

CONCLUSIONS

The study determined that SLDs are used to assess the stability limits of the milling process, which determine the critical depth of cut for each rotational speed. Traditionally, methods based on DDE equations and analysis of Lyapunov exponents are used to construct SLDs. For complex systems, such as multimodal or nonlinear rotational speeds, advanced methods (SDM, Galerkin, Time-Domain Simulation) are used. The results of the study showed the effectiveness of stabilisation methods. 3D SLD with position dependence considered the change in stiffness along the trajectory, which is substantial for machining thin-walled structures in the aerospace industry. NDDE + TVA provided an effective analysis of mode and regenerative self-oscillations in the machining of flexible titanium components. For simpler cases, such as machining aluminium or steel parts, the $\omega(t)$ rotation modulation method is effective. In 6DOF robotic systems that incorporate the flexibility of the manipulator, methods of adaptation to changing conditions were used. The DRL can be used to adapt the system to changing parameters and generate signals for piezoelectric actuators to effectively suppress vibrations in real time.

A comparative analysis of the models showed that each approach has its advantages and limitations. 3D SLD provides high accuracy but requires significant computing resources. NDDE + TVA is effective for chaotic modes but is difficult to implement. Modulation $\omega(t)$ is easy to implement, but less effective for complex geometries. Systems with 6DOF demonstrate high dynamism, but their accuracy depends on the quality of the models. Probabilistic models optimally incorporate parameter fluctuations but are resource intensive. To achieve maximum accuracy, it is recommended to combine analytical and numerical methods with experimental measurements. Adaptability is critical for processing different materials and complex geometries. DRLs and robotic systems with 6DOF are a promising direction for high-precision milling operations.

A limitation of the study was that the models did not fully incorporate the effects of wear and thermomechanical instability, which could be substantial in long machining cycles. This created opportunities for future research, in particular in creating complex digital models with sensory feedback and self-learning mechanisms. Further research should facilitate the identification of parameters and the integration of active stabilisation methods into automated systems to improve milling stability and productivity.

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CONFLICT OF INTEREST

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Вплив реґенеративного ефекту та модового зчеплення на стабільність фрезерних операцій

Анотація. Забезпечення якості фрезерної обробки потребує врахування динамічної стабільності процесу, що залежить від інструментальної геометрії, конструктивної жорсткості та типу вібрацій. Мета дослідження була спрямована на виявлення впливу внутрішніх динамічних збурень на перебіг фрезерного процесу, зокрема механізмів самозбуджених вібрацій, що виникають через коливальні взаємодії та залишкові нерівності обробленої поверхні. Методологія включала порівняльний аналіз моделей стабілізації фрезерування із оцінкою їх переваг, обмежень та практичних рекомендацій для впровадження в реальні виробничі процеси. Було представлено математичні моделі, що враховували ефекти затримки у часі та взаємодії між напрямками коливань інструмента і заготовки. Встановлено, що повторний контакт інструмента з нерівностями попередньо обробленої поверхні призводить до змінної товщини шару матеріалу формував нестійкий режим роботи. Взаємодія між різними коливальними режимами, при обробці гнучких або тонкостінних деталей, призводить до більш складних автоколивальних процесів, включаючи хаотичні коливання. Найбільш перспективними для практичного застосування є моделі з адаптивним керуванням, здатні реагувати на зміну технологічних параметрів у реальному часі. Встановлено, що динаміка фрезерного процесу суттєво залежить від геометрії інструмента, довжини виступу, жорсткості, масоінерційних характеристик та зношення ріжучої частини. Процесне демпфування, спричинене бічним контактом інструмента з деталлю, зменшує амплітуду коливань, в діапазоні низьких частот. Структура керування на основі вектора стану системи включала положення, швидкість, амплітуду вібрацій, оберти шпинделя та подачу інструмента. Жоден з методів не є універсальним, тому доцільно застосувати адаптивний підхід до вибору моделі стабільності з урахуванням конструктивних, динамічних і економічних факторів. Практична цінність результатів полягала у можливості застосування їх інженерами та технологами для впровадження адаптивного керування фрезеруванням для зменшення вібрацій і підвищення точності обробки

Ключові слова: автоколивання; система із запізненням; багатомодова динаміка; напівдискретизація; вектор стану; інтелектуальне керування; адаптивне моделювання