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Synthesis of the optimal acceleration mode of the tower crane slewing mechanism

Abstract. The high performance of cranes is one of the main requirements for construction of buildings and constructins. Tower cranes usually perform such operations. One of the most effective ways to achieve this requirement is to eliminate pendulum oscillations of the load on the flexible suspension for tower cranes in the shortest possible time. Therefore, this article aimed to obtain a general solution to the problem of optimal performance control during the acceleration of the crane's slewing mechanism. A dynamic system model of the tower crane was presented, boundary conditions and optimisation criteria were described, and a torque constraint was introduced. The original problem was reduced to the problem of unconstrained optimisation of a nonlinear function. The problem was solved multiple times, in which the mass of the load m, the length of the flexible suspension l, and the boom extension range r were varied within certain limits. The VCT-PSO optimisation algorithm was used to find solutions. Based on the obtained solutions, a data set for training the artificial neural network (training set) and a data set for testing its operation (test set) were formed. There were used a feedforward neural network to solve the problem of predicting the moments of control switching. Three inputs (corresponding to the parameters m, r, l), three outputs (corresponding to the moments of control switching), and three hidden layers with five neurons in each were used. The neural network was trained using the ADAM method. The results of estimating the quality of the prediction of the control switching moments were determined, and the plots of estimation errors were presented. The high quality of the approximation of the solution surface of the problem using the artificial neural network was established. The analysis of the system motion dynamics under optimal control was carried out, which demonstrated that the system motion indicators were least affected by the mass of the load m and most affected by the length of the flexible suspension l

Keywords: tower crane; slewing mechanism, optimal control; pendulum oscillations of the load; artificial neural network

INTRODUCTION

People widely use tower cranes for load movement on construction sites. To enhance the efficiency of tower cranes, they must quickly move load while eliminating load oscillations on a flexible suspension at the end of the transitional movement mode (acceleration/braking). Many researchers have worked on developing optimal modes of motion for this mechanism. In work by V.O. Kovalenko (2021) the problem of optimal control of tower crane rotation was set

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to minimise the duration of the work cycle. This problem was solved using the principle of maximum (Pucci, 2004; Kiianovskyi & Kiyanovska, 2022). According to V.V. Svirgun (2022) a structural system diagram for optimal control of the mechanisms for changing the load outreach and crane rotation was developed, along with recommendations regarding its hardware part. In practice, optimal performance control laws were implemented, including eliminating load oscillations after the crane stops and precise load positioning with the minimum number of control stages in the shortest timeIn the work by Y.M. AI-Rawashdeh (2023), a framework was presented that enables designing kinodynamical reference trajectories and detecting the flexible modes of uncertain systems while in motion. In the work by A.O. Okun (2021) the problem of optimising load movement of a load on a flexible suspension over a given distance was considered. To construct the control algorithm for the "trolley-load" system, phase plane methods using poles of corresponding transfer functions were applied. In work of O.O. Kovalenko *et al.* (2022) the problem of identifying control laws for the slewing mechanism of a jib crane was solved using the maximum principle. An algorithm for finding optimal performance control was found and described. The law of optimal movement of the slewing mechanism of a laboratory jib crane was calculated.

Zh. Liu *et al.* (2021) presented the development of an algorithm to eliminate the oscillations of a ship crane based on a constructed mathematical model. Using the AMEsim-ADAMS platform, the problem was solved. It simulates the load oscillations of a ship crane in simulated deck movement conditions. H. Ouyang *et al.* (2021) researched controlling rotary crane systems. A method for implementing the positioning of the boom and eliminating the oscillation of the moving load by a rotary crane was proposed. The presented boom positioning trajectory can be calculated online. In the study by Z. Yu *et al.* (2021) optimising bridge crane control using a fuzzy PID controller was considered. The researchers proved that the fuzzy PID controller surpasses the traditional PID controller in positioning accuracy, elimination of load oscillations, and efficiency of obstacle avoidance.

This paper aimed to design the optimal time-efficient control mode for the tower crane slewing mechanism. The mandatory condition, which was associated with needed optimal control, was the elimination of load pendulum oscillations. The originality of the presented work is a new approach to optimal control design, which is based on approximation features of an artificial neural network (ANN). It, in turn, allows to generate of optimal control for a wide range of values of dynamical system "crane-load" parameters. Thus, ANN may be considered as a solver of the optimal control problem in a general sense.

MATERIALS AND METHODS

For the research, a two-mass dynamic model of the crane slewing mechanism was adopted (Fig. 1) (Kang & Miranda, 2004).

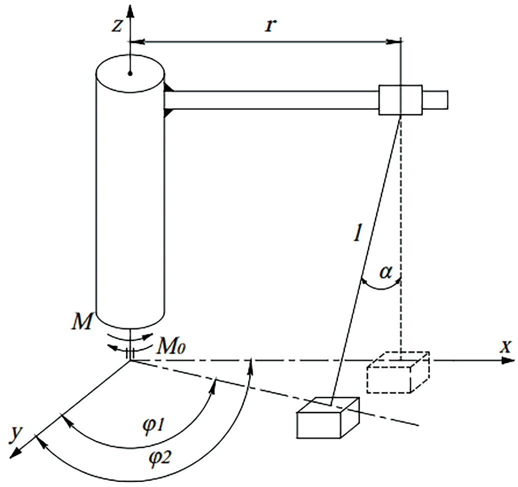

Figure 1. Dynamic model of the jib crane slewing mechanism

Note: M_0 – the combined resistance torque that arises in the crane rotary part; M – the combined motor torque driving the slewing mechanism of the crane; m – the mass of the load; l – the length of the cable; r – the length of the crane's boom; I – the moment of inertia of the elements of the drive mechanism of slewing mechanism and the rotating tower, reduced to the crane rotary axis; ϕ_1 – angular coordinate of crane slewing; ϕ_2 – angular coordinate of load position; x, y, and z – Cartesian coordinates

Source: compiled by the authors

The generalised coordinates of the dynamic model were the angular coordinates of rotation, reduced to the rotation axis of the crane elements of the slewing mechanism drive and the rotating tower with the boom ϕ_1 and the load ϕ_2. Thus, in the study, the angular system was the reference. All torques that occur in this system were taken into account. Others (for instance, torques in the plane of the crane) were neglected. The equations of motion corresponding to this model were taken from the work with condition C→∞:

$$\begin{cases} M - M_0 sign(\dot{\phi}_1) = I\ddot{\phi}_1 + mr^2\frac{g}{l}(\phi_1 - \phi_2); \\ \ddot{\phi}_2 = \frac{g}{l}(\phi_1 - \phi_2), \end{cases} \tag{1}$$

where M_0 – the combined resistance torque that arises in the crane rotary part; M – the combined motor torque driving the slewing mechanism of the crane; m – the mass of the load; l – the length of the cable; r – the length of the crane's boom; I – the moment of inertia of the elements of the drive mechanism of slewing mechanism and the rotating tower, reduced to the crane rotary axis. For the process of starting the slewing mechanism of the jib crane, the following boundary conditions were used:

$$\begin{cases} \phi_1(0) = \dot{\phi}_1(0) = \phi_2(0) = \dot{\phi}_2(0); \\ \phi_1(T) = \phi_2(T) = \dot{\phi}_1(T) = \dot{\phi}_2(T) = \omega, \end{cases} \tag{2}$$

where ω – the rated angular velocity of the crane slewing. The initial conditions mean the system's movement from a state of rest, i.e., when there is no movement. The final conditions meant the system's movement at the nominal angular velocity ω in the absence of pendulum oscillations of the load. To conduct optimisation, a performance criterion was used, reflecting the duration of the crane slewing mechanism acceleration:

$$\int_0^T dt \to min = T,\ T = t_1 + t_2 + t_3, \tag{3}$$

where T – the duration of the system movement; t_1, t_2, t_3 – the durations of the first, second, and third stages, respectively. The engine that drives the slewing mechanism of the crane was characterised by its overload capacity, so constraints on the torque were introduced:

$$M_{\min} \leqslant M \leqslant M_{\max}, \tag{4}$$

where $M_{\min}$ and $M_{\max}$ – the minimum and maximum values of the driving torques. The asynchronous motor, which is part of the drive of the slewing mechanism of the crane, is characterised by symmetry, hence, the following expression was valid: $M_{\min} = -M_{\max}$. A constraint on the angular velocity of the crane's rotationW was introduced:

$$0 \leqslant \dot{\phi}_1 \leqslant \omega, \tag{5}$$

where the angular velocity must be within the range from 0 to ω. This allowed acceleration to the nominal speed without exceeding it. This, in turn, enabled the rotation of the crane without additional energy expenditure. Since the movement is in one direction, a constraint was introduced that the angular velocity should not be less than zero.

Problems (1)-(5) are pretty complex. Therefore, it was reduced to a problem of unconstrained optimisation. The second equation of the system (1) was twice differentiated concerning time:

$$\ddot{\phi}_1 = \ddot{\phi}_2 + \frac{l}{g}\overset{IV}{\phi}_2. \tag{6}$$

The substitution of expression (6) into the first equation of system (1) gave:

$$M - M_0 = I(\ddot{\phi}_2 + \frac{l}{g}\overset{IV}{\phi}_2) + mr^2\frac{g}{l}(\phi_1 - \phi_2). \tag{7}$$

Taking into account the second equation of system (1), the equation (7) was rewrited as follows:

$$M - M_0 = I\left(\ddot{\phi}_2 + \frac{l}{g}\overset{IV}{\phi}_2\right) + mr^2\ddot{\phi}_2 =$$
$$= I\phi_2 + \frac{l}{g}I\overset{IV}{\phi}_2 + mr^2\ddot{\phi}_2 = \frac{l}{g}I\overset{IV}{\phi}_2 + (I + mr^2)\ddot{\phi}_2. \tag{8}$$

Both sides of expression (8) were divided by the coefficient near the highest derivative of the function ϕ_2:

$$\frac{M - M_0}{I}\cdot\frac{g}{l} = \overset{IV}{\phi}_2 + \frac{I + mr^2}{I}\cdot\frac{g}{l}\ddot{\phi}_2. \tag{9}$$

The notations were introduced:

$$\Omega_0 = \sqrt{\frac{g}{l}}; \tag{10}$$

$$\Omega = \sqrt{\frac{I + mr^2}{I}\cdot\frac{g}{l}};\ M - M_0 = M_{din}. \tag{11}$$

The expression (10) can be rewritten in the following form:

$$\overset{IV}{\phi}_2 + \Omega^2\ddot{\phi}_2 = \frac{M_{din}}{I}\cdot\Omega_0^2. \tag{12}$$

The acceleration period can be considered a movement in three stages, where the torque M takes a constant value ($M_{\max}$ or $M_{\min}$). This fact allowed obtaining an analytical solution of the differential equation (12) for the i-th stage:

$$\begin{aligned}\phi_2 = \frac{1}{2I\Omega^2}(\ddot{\phi}_2(\tau_{i-1}) + \dddot{\phi}_2(\tau_{i-1})(t - \tau_{i-1}) + (\ddot{\phi}_2(\tau_{i-1})(t - \tau_{i-1}) +\\ +2\,(\tau_{i-1})\Omega^2) + M_{din}(-2 + (t - \tau_{i-1})^2\,\Omega_0^2 + (-2\ddot{\phi}_2(\tau_{i-1}))m\Omega^2 +\\ +2M_{din}\Omega_0^2)cos((t - \tau_{i-1})\Omega) - 2\dddot{\phi}_2 m\Omega sin((t - \tau_{i-1})\Omega));\\ \tau_0 = 0,\ \tau_1 = t_1,\ \tau_2 = t_1 + t_2,\end{aligned} \tag{13}$$

where τ_{i-1} – the moment of the beginning of the i-th stage of the dynamic system's movement.

The next step in the calculations was to determine the function φ_2 for the system's first, second, and third acceleration stages using equation (13).

$$\begin{cases}\phi_2(\tau_0) \to 0,\ \dot{\phi}_2(\tau_0) \to 0,\ \ddot{\phi}_2(\tau_0) \to 0,\ \dddot{\phi}_2(\tau_0) \to 0,\\ M_{din} \to (M_{max} - M_0),\ \tau_0 \le t \le \tau_1;\\ \phi_2(\tau_1) \to \phi_{2t_1}, \dot{\phi}_2(\tau_1) \to \dot{\phi}_{2t_1}, \ddot{\phi}_2(\tau_1) \to \ddot{\phi}_{2t_1}, \dddot{\phi}_2(\tau_1) \to \dddot{\phi}_{2t_1},\\ M_{din} \to (M_{max} - M_0),\ \tau_1 \le t \le \tau_2;\\ \phi_2(\tau_2) \to \phi_{2t_1}, \dot{\phi}_2(\tau_2) \to \dot{\phi}_{2t_1+t_2}, \ddot{\phi}_2(\tau_2) \to \ddot{\phi}_{2t_1+t_2}, \dddot{\phi}_2(\tau_2) \to \dddot{\phi}_{2t_1+t_2},\\ M_{din} \to (M_{max} - M_0),\ \tau_2 \le t \le T.\end{cases} \tag{14}$$

As a result, three solutions corresponding to the three stages of the system's acceleration were obtained. Let us denote them $\varphi_{2.1}$, $\varphi_{2.2}$, $\varphi_{2.3}$ (the indices 1, 2 and 3 reflect the corresponding stage number). Having all the solutions for the three stages, the conditions for combining the solutions at the moments τ_1 and τ_2 were set:

$$\begin{cases}\phi_{2.1}(\tau_1) = \phi_{2\tau_1}, \dot{\phi}_2(\tau_1) = \dot{\phi}_{2\tau_1}, \ddot{\phi}_2(\tau_1) = \ddot{\phi}_{2\tau_1}, \dddot{\phi}_2(\tau_1) = \dddot{\phi}_{2\tau_1};\\ \phi_{2.2}(\tau_2) = \phi_{2\tau_2}, \dot{\phi}_2(\tau_2) = \dot{\phi}_{2\tau_2}, \ddot{\phi}_2(\tau_2) = \ddot{\phi}_{2\tau_2}, \dddot{\phi}_2(\tau_2) = \dddot{\phi}_{2\tau_2}.\end{cases} \tag{15}$$

These calculations allowed finding the function $\varphi_{2.3}(t)$. It was not provided here due to its significant volume. Substituting $t \to t_1 + t_2 + t_3$ into the function $\varphi_{2.3}(t)$ allowed to obtain the magnitude of $\varphi_{2.3}(T)$, and the magnitude $\dot{\phi}_{2.3}$ (T), $\ddot{\phi}_{2.3}(T)$, $\dddot{\phi}_{2.3}(T)$. They can be presented in the following form:

$$\begin{aligned}\varphi_{2.3}(T) = \frac{1}{2I\Omega^4}\Omega_0^2(2M_0 - 2M_{\max} - M_0(t_1 + t_2 + t_3)^2\Omega^2 +\\ + M_{\max}(t_1^2 - t_2^2 - 2t_2t_3 + t_3^2 + 2t_1(t_2 + t_3))\Omega^2 + 4M_{\max}\cos(t_3\Omega) -\\ - 4M_{\max}\cos((t_2 + t_3)\Omega) + 2(-M_0 + M_{\max})\cos((t_1 + t_2 + t_3)\Omega));\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{\varphi}_{2.3}(T) = \Omega_0^2(M_{\max}(t_1 - t_2 + t_3)\Omega - M_0(t_1 + t_2 +\\ + t_3)\Omega - 2M_{\max}\sin(t_3\Omega) + 2M_{\max}\sin((t_2 + t_3)\Omega) +\\ + (M_0 - M_{\max})\sin((t_1 + t_2 + t_3)\Omega));\end{aligned} \tag{17}$$

$$\begin{aligned}\ddot{\varphi}_{2.3}(T) = \Omega_0^2(-2M_{\max}\cos(t_3\Omega) + 2M_{\max}\cos((t_2 + t_3)\Omega) +\\ + (M_0 + M_{\max})(-1 + \cos((t_1 + t_2 + t_3)\Omega));\end{aligned} \tag{18}$$

$$\ddot{\varphi}_{2.3}(T)=\Omega_0^2(2M_{\max}\sin(t_3\Omega)-2M_{\max}\sin((t_2+t_3)\Omega)+$$
$$+(-M_0+M_{\max})\sin((t_1+t_2+t_3)\Omega)). \qquad (19)$$

Taking into account expressions (16)-(19), the conditions that correspond to the final conditions of the system's movement were writen: its acceleration to the nominal angular velocity *W* and ensuring the absence of pendulum oscillations of the load at the end of acceleration. They were represented in the form of a system of transcendental algebraic equations:

$$\begin{cases} \omega-\dot{\phi}_{2.3}(T)=0; \\ \ddot{\phi}_{2.3}(T)=0; \\ \dddot{\phi}_{2.3}(T)=0. \end{cases} \qquad (20)$$

Again, due to the significant volume of the obtained expressions, they were not presented in an expanded form.

To solve the system of transcendental algebraic equations (20), the TE function were construct, which in a certain sense reflects the level of satisfaction of the final conditions (2):

$$TE=\frac{mr^2}{2}\left(\left(\omega-\dot{\varphi}_{2.3}(T)\right)^2+\left(\ddot{\varphi}_{2.3}(T)\right)^2+\left(\frac{\dddot{\varphi}_{2.3}(T)}{\Omega_0^2}\right)^2\right)=$$
$$=\frac{1}{2I^2\Omega^6}mr^2(\Omega^2\Omega_0^4(-2M_{\max}\cos(t_3\Omega)+2M_{\max}\cos((t_2+t_3)\Omega)+$$
$$+(M_0-M_{\max})(-1+\cos((t_1+t_2+t_3)\Omega)))^2+\Omega^4(2M_{\max}\sin(t_3\Omega)-$$
$$-2M_{\max}\sin((t_2+t_3)\Omega)+(-M_0+M_{\max})\sin((t_1+t_2+t_3)\Omega))^2+$$
$$+((-M_{\max}(t_1-t_2+t_3)+(M_0(t_1+t_2+t_3))\Omega\Omega_0^2+2M_{\max}(\Omega^2-$$
$$-\Omega_0^2)\sin(t_2\Omega)+(M_0-M_{\max})(\Omega^2-\Omega_0^2)\sin((t_1+t_2)\Omega). \qquad (21)$$

The first term in brackets in expression (21) corresponded to the kinetic 'excess or insufficient energy' of mass *m*. It showed the deviation of the energy of mass *m* at the moment $t_1+t_2+t_3$ from the value $m\omega^2/2$. The second and third terms in expression (21) were the potential and kinetic energy of the load's (pendulum's) oscillations at the end of the system's acceleration.

Thus, the initial problem (1)-(5) was reduced to the following:

$$\underset{t_1,t_2,t_3}{argmin}\left(\frac{2E}{m\omega^2}+(t_1+t_2+t_3)\frac{\Omega}{2\pi}\right),$$
$$E=\begin{cases} w\cdot TE,\ if\ \ TE\geq\Delta; \\ 0,\ if\ \ TE<\Delta, \end{cases} \qquad (22)$$

where Δ – a certain conditional energy threshold corresponding to the deviation of the system's state from the desired one at the end of its acceleration, i.e., it is a function that defines the deviation of the final phase coordinates from the final conditions (2): the smaller the Δ, the closer the final phase coordinates are to the final conditions (2) (within this study $\Delta=10^{-2}$ J); *w* – a weight coefficient that ensures the importance of minimising TE (set to $w=10^6$); *E* – a nonlinear function that reflects the fulfillment of the condition $TE<\Delta$.

The first term in brackets in expression (22) was the system's specific "excess" energy at the end of the system's acceleration. The second was the specific duration of the system's acceleration. Both addends are dimensionless and, therefore, can be added. The exact solution of problem (1)-(5) reduces the first term of criterion (22) to a value that was less than the threshold Δ, when all the final conditions (2) were practically fulfilled. Including the values of *w* and Δ in the objective function (22) can be explained by a two-stage process of searching for a solution to the problem. In the first stage, the value of *E* was minimised. The optimisation algorithm searched the minimum *T* (since $E=0$) in the second stage. Such a structure of the objective function (22) allowed for the avoidance of false solutions to the original problem (2)-(4). Indeed, at least two such cases can be imagined:

1) $w=1$, then $t_1\to 0$, $t_2\to 0$, $t_3\to 0$. In this case, the objective function (22) has a global minimum, which equals 1. The second and third terms of *TE* in brackets (the first line of expression (21)) equaled zero, and only the first term equaled ω;

2) $\Delta=0$, then at some stage of minimising the objective function (22), the product $w\cdot TE$ was reduced to a value comparable to t_1, t_2 and t_3 values. This meant that the requirements for minimising the magnitudes of $w\cdot TE$ and $(t_1+t_2+t_3)\Omega/2\pi$ start competing. In this case, there was no guarantee of a complete minimisation of the criterion (3).

To ensure compliance with condition (5), the potentially highest speed of crane rotation during acceleration was found, which was at the end of the first stage when the tower begins to accelerate:

$$\dot{\varphi}_2(t_1)=-\frac{1}{I\Omega^3}(M_{\max}(-t_1+t_2)+M_0(t_1+t_2))\Omega\Omega_0^2+2M_{\max}(\Omega^2-$$
$$-\Omega_0^2)\sin(t_2\Omega)+(M_0-M_{\max})(\Omega^2-\Omega_0^2)\sin((t_1+t_2)\Omega). \qquad (23)$$

The potentially lowest speed of crane rotation during acceleration was found, which was at the end of the second stage when the tower was braking:

$$\dot{\phi}_2(t_1+t_2)=-\frac{(M_0-M_{max})\left(t_1\Omega\Omega_0{}^2+(\Omega^2-\Omega_0{}^2)\right)\sin\,(t_1\Omega)}{I\Omega^3}. \qquad (24)$$

Expression (23) was found to fulfill the constraint of not exceeding ω, and expression (24) was found to fulfill the constraint of no crane reversal (absence of negative angular velocity of crane rotation). Further, to take into account the constraints (5), penalty functions were constructed:

$$P_1=\begin{cases} \frac{\dot{\phi}_2(t_1)}{\omega}\cdot\lambda_1,\ if\ \dot{\phi}_2(t_1)>\omega; \\ 0, if\,\dot{\phi}_2(t_1)\leq\omega, \end{cases} \qquad (25)$$

$$P_2=\begin{cases} \frac{\dot{\phi}_2(t_1+t_2)}{\omega}\cdot\lambda_2,\ if\ \dot{\phi}_2(t_1+t_2)<0; \\ 0, if\,\dot{\phi}_2(t_1+t_2)\geq 0, \end{cases} \qquad (26)$$

where λ_1, λ_2 – weight coefficients determining the importance of fulfilling the constraints (5). If the upper or lower constraints were violated, functions P_1, or P_2 took large values. Their magnitudes were regulated precisely by the weight coefficients λ_1, λ_2, which in this study were equal to 10^8. Thus, the problem (1)-(5) was reduced to the minimisation of the following function:

$$\underset{t_1,t_2,t_3}{argmin}\left(\frac{2E}{m\omega^2}+(t_1+t_2+t_3)\frac{\Omega}{2\pi}+P_1+P_2\right). \quad (27)$$

It was necessary to set numerical values for the system to solve the problem (27) (Table 1).

Table 1. Parameters of the "crane-load" system

Parameter	Unit of measurement	Value
m	kg	200-5000
l	m	3-15
r	m	2.5-40
ω	rad/s	0.07
M_{max}	Nm	257142
M_0	Nm	25714
I	kg·m²	4992364

Source: compiled by the authors

The values of m, l, r in Table 1 were given as ranges. This was necessary to reflect the practical side of the tower crane operation. Its work occured with different masses of load m, different lengths of flexible suspension l, and different outreach of load r. The limits of the ranges for these values, as was shown in Table 1, correspond to the maximum and minimum values encountered in operating a tower crane.

RESULTS AND DISCUSSION

To solve problem (27), the VCT-PSO optimisation algorithm was used, which is a profound modification of the canonical Particle Swarm Optimisation (PSO) method (Romasevych & Loveikin, 2022). The following settings of this algorithm were used: $c_1 = c_2 = 1.19$, RC=м5, $w = 0.72$. Example plots corresponding to the results of solving the problems using the VCT-PSO algorithm were given below. For example, Figure 2 presented a plot of the convergence of function (27) when solving the problem for the following parameters: $m = 600$ kg, $l = 10$ m, $r = 25$ m.

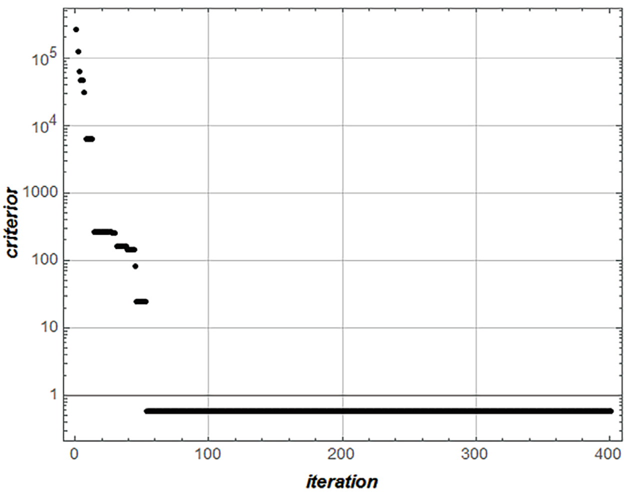

Figure 2. Plot of convergence of function (27) for parameters $m = 600$ kg, $l = 10$ m, $r = 25$ m

Source: compiled by the authors

As can be seen from the plot, the value of the criterion decreased during the first seventy iterations, and from the seventieth to the four-hundredth iteration, function (27) remained unchanged. Overall, this algorithm found solutions for three thousand five hundred ninety-seven problems (for different variants of load mass m, length of flexible suspension l and load outreach r). All of them were presented in the following format: $\{t_1, t_2, t_3, m, r, l\}$. The found solutions were used for training an artificial neural network, which was used to approximate the entire set of solutions to problems (1)-(5).

Calculation of optimisation problem solution each time required a large amount of computation resources. To make the process of values t_1, t_2, t_3 mining much quicker (and implemented in crane control system) an artificial neural network was used. After its training and testing it may be exploited to calculate values t_1, t_2, t_3 for different practical cases of exploitation (varying values m, r, l). From the found solutions to the problems, a data array was formed for training the artificial neural network (training sample) and an array for testing its performance (test sample). Overall, 2886 solutions were selected for training the artificial neural network and 711 for its testing.

This study used a feedforward neural network with 3 inputs (corresponding to parameters m, r, l), 3 outputs (corresponding to values t_1, t_2, t_3) and 3 hidden layers with 5 neurons each.

Each neuron of the artificial neural network was characterised by a "hyperbolic tangent" activation function. The neural network was fully connected, and each neuron received a bias signal. To conduct the training of the artificial neural network, data normalisation was carried out:

$$\tilde{m} = \frac{m-m_{min}}{m_{max}-m_{min}}; \quad \tilde{r} = \frac{r-r_{min}}{r_{max}-r_{min}}; \quad \tilde{l} = \frac{l-l_{min}}{l_{max}-l_{min}}, \quad (28)$$

where $\tilde{m}$, $\tilde{r}$, and $\tilde{l}$ – the normalised values of the load mass, boom outreach, and the length of the flexible suspension, respectively. Normalised values range was from zero to one.

The neural network training was carried out using the ADAM method (Kingma & Ba, 2015). The batch size equals 500. How accurately the artificial neural network approximates data needs to be determined. Thus, the RMS error (RMSE) of the prediction of the duration of the system's movement stages was calculated according to the formula:

$$e_{RMS.t1} = \sqrt{\sum_{i=1}^{I}\left(\frac{E_{t1.i}(t_{1.\max}-t_{1.\min})}{t_{1.m.i}(t_{1.\max}-t_{1.\min})+t_{1.\min}}\right)^2}\,100\%, \qquad (29)$$

where $e_{RMS.t1}$ – the RMS error of the prediction of the duration of the first stage of the system's movement; $E_{t1.i}$ – the i-th error of the prediction of the duration of the first stage of the system's movement; $t_{1.\text{т}.i}$ – the i-th value of the duration of the first stage of the system's movement from the test data sample; $t_{1.\max}$ – the maximum value of the duration of the first stage of the system's movement across the entire test sample; $t_{1.\min}$ – the minimum value of the duration of the first stage of the system's movement across the entire test sample; I – the length of the test data sample. Similar calculations were used to estimate the durations of the second and third stages. In addition to the RMS indicators, the maximum errors were also used:

$$e_{\max.t1} = \max\left(\left|\frac{E_{t1.i}(t_{1.\max}-t_{1.\min})}{t_{1.m.i}(t_{1.\max}-t_{1.\min})+t_{1.\min}}\right|\right)100\%, \qquad (30)$$

where $e_{\max.t1}$ – the maximum error in predicting the duration of the first stage of the system's movement. Finding the maximum errors for the second and third stages was carried out with using similar formulas.

The obtained error values were entered into Table 2. Table 2 shows that the values of the RMS errors were insignificant and approximately within one percent. The maximum errors of the 1-st and 3-rd stages (1.44 and 1.63%, respectively) were also minor. Only the maximum error of the second stage stands out at 7.34%.

Table 2. Results of the quality assessment of the prediction of the duration of the dynamic system's movement stages performed by the artificial neural network

Stage number	Errors, %	
	RMS	Maximal
1st	0.34	1.44
2nd	1.08	7.34
3rd	0.41	1.63

Source: compiled by the authors

For clarity of the obtained estimates, Figure 4 presented the plots of the error in the duration of the system's movement (separately for each acceleration stage). The index j indicates the test frame number used for the evaluation. All the calculations that assessed the dynamics of the system movement under optimal control were conducted using eight parameter variations (Table 3). In Table 3, the following notations were used: $m_{\min}$ = 300 kg, $m_{\max}$ = 4500 kg, $r_{\min}$ = 3.75 m, $r_{\max}$ = 36 m, $l_{\min}$ = 4.5 m, $l_{\max}$ = 13.5 m.

Table 3. Variants of parameter combinations for the assessment of movement dynamics under optimal control

<table>
<tr><th rowspan="2">Variant Number</th><th colspan="3">Parameter</th></tr>
<tr><th>m</th><th>r</th><th>l</th></tr>
<tr><td>1</td><td rowspan="4">$m_{\min}$</td><td rowspan="2">$r_{\min}$</td><td>$l_{\min}$</td></tr>
<tr><td>2</td><td>$l_{\max}$</td></tr>
<tr><td>3</td><td rowspan="2">$r_{\max}$</td><td>$l_{\min}$</td></tr>
<tr><td>4</td><td>$l_{\max}$</td></tr>
<tr><td>5</td><td rowspan="4">$m_{\max}$</td><td rowspan="2">$r_{\min}$</td><td>$l_{\min}$</td></tr>
<tr><td>6</td><td>$l_{\max}$</td></tr>
<tr><td>7</td><td rowspan="2">$r_{\max}$</td><td>$l_{\min}$</td></tr>
<tr><td>8</td><td>$l_{\max}$</td></tr>
</table>

Source: compiled by the authors

Calculations were carried out for each of the eight variants using the already trained artificial neural network. For example, a graphical interpretation of the solutions to problems (1)-(5) only for the first (Fig. 3) were presented. All values on the graphs were normalised, that was, the maximum value was equal to 1, and the minimum value was -1.

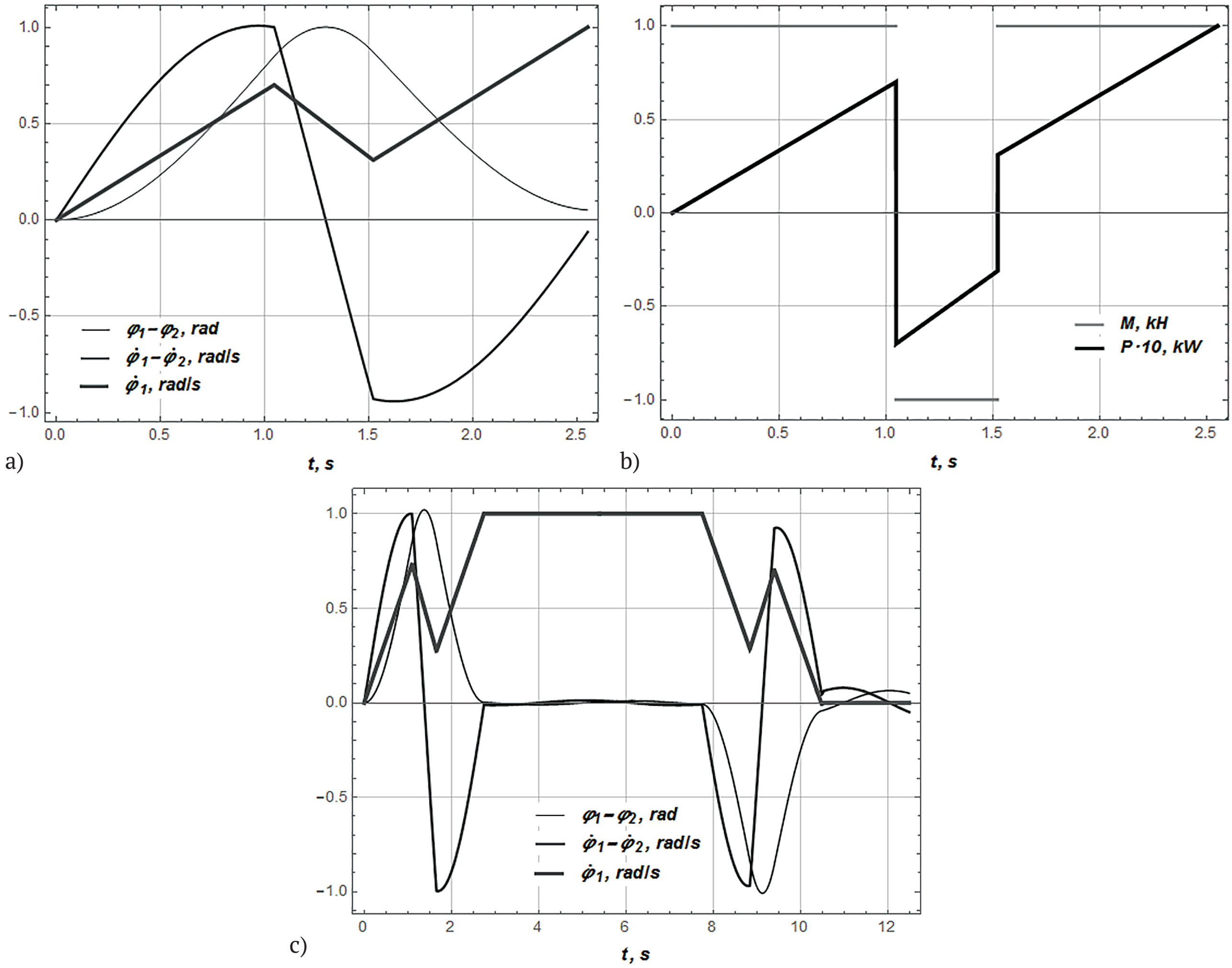

Figure 3. Plots of kinematic, dynamic, and energy characteristics of the dynamic system's movement corresponding to the first number of parameter variants (Table 3)

Note: a) the angle of cable deviation from the vertical and its first time derivative, as well as the crane's rotation speed during acceleration; b) the driving torque of the crane's drive and the power consumed by the drive during acceleration; c) the angle of cable deviation from the vertical and its first time derivative, as well as the crane's rotation speed during the cycle "acceleration-steady movement – deceleration" and after it

Source: compiled by the authors

Figure 3a presents three plots. The bold black line shows the angular velocity of the system, which was represented as a broken line because during the period from 1.05 seconds to 1.50 seconds, the system brakes, and then after some time, it increases its speed again. The thin black line represented the difference in the speeds of the crane and the load's motion. The gray line represented the difference in the angular positions of the crane and the load.

Figure 3b showed a plots of the system's force characteristics, where the black line represented the power of the crane's slewing mechanism drive during acceleration. As noted above, the system breaks from 1.05 seconds to 1.50 seconds, so the power plots changed sign to the opposite. At this point, the drive operates in a regenerative mode, i.e., it changed its operating mode: instead of working as a motor, it started working as a generator, producing energy. It is necessary to use a specific frequency converter to implement such a mode. The converter has to recuperate energy into the network (with a reverse inverter).

In order to prove the efficiency of the optimal control in the sense of pendulum oscillations elimination, Figure 3c gave the functions of the system movement during the cycle "acceleration-steady movement – deceleration" and after it (during 5 s). It clearly shows almost complete elimination of oscillations. In the plot in Figure 3c are caused by the fact that only the minimum values of the load mass, the length of the flexible suspension, and the boom outreach were taken for calculation. An analysis of the assessment indicators of the system's motion under optimal control for each of the eight variants was also carried out, which are presented in Table 4. Analysing the data in Table 4, brought that the duration of the first and third stages have almost identical values.

Table 4. Assessment indicators of the dynamics of optimal control of the "crane-load" system's movement

Indicator	Variant number							
	1	2	3	4	5	6	7	8
Duration of the 1-st stage, s	1.045	1.477	1.063	1.504	1.061	1.502	1.121	1.532
Duration of the 2-nd stage, s	0.481	1.182	0.470	1.172	0.474	1.176	0.395	1.115
Duration of the 3-rd stage, s	1.026	1.465	1.058	1.498	1.057	1.495	1.115	1.530
The amplitude of the angle of cable deviation from the vertical, rad	0.023	0.062	0.024	0.063	0.024	0.063	0.023	0.063
The maximum difference in rotation speeds of the crane and the load, rad/s	0.029	0.054	0.029	0.055	0.028	0.055	0.027	0.054
The maximum power in reverse mode (at the beginning of the second stage), kW	12.45	17.56	12.60	17.88	12.55	17.83	12.36	17.61
The RMS of the angle of cable deviation from the vertical, rad	0.013	0.035	0.013	0.036	0.013	0.036	0.013	0.035
The RMS of the difference in rotation speeds of the crane and the load, rad/s	0.021	0.033	0.021	0.034	0.021	0.034	0.021	0.034
The RMS of the crane drive's power, kW	9.774	10.27	10.07	10.59	9.941	10.48	9.892	10.11
RMS of the drive torque of the crane, kNm	257.1							

Source: compiled by the authors

When analysing the duration across all 3 stages, it was evident that the values of the duration of individual stages in the odd variants were less than those corresponding to the even variants. This can be explained by the fact that different lengths of flexible suspension were used for these variants. The amplitude of the angle of cable deviation from the vertical for even variants have much bigger values, than those, which reflected to the odd variants. The same may be stated (but to a somewhat lesser extent) for the maximum difference in rotation speeds of the crane and the load. The reason of this outcome may be found in Table 3. Indeed, in the even variants l_{max} value was involved, for the odd ones – l_{min}. Thus, the more l value, the more amplitudes of the load oscillations. This conclusion may be generalised to the RMS values (Table 4) as well. RMS of the drive torque of the crane is the same for all variants because the crane drive was operated at the extreme (maximum or minimum) torque throughout the duration of the system's acceleration.

One advantage of the fuzzy PID controller is its insensitivity to system internal parameter changes. In work Yu. Qian *et al.* (2021) the adaptive control of a marine crane based on a neural network was described. The developed algorithm improves the efficiency of obstacle avoidance and provides high-performance control, accurate load positioning, and quick reduction of pendulum oscillations of the load. In work by Yu. Romasevych *et al.* (2022) the synthesis of optimal control of the "crane-load" system was considered. The article examined problem of optimal control of the system, which ensures the elimination of load oscillations, minimisation of energy consumption, and maximum crane work productivity. The problem was solved by minimising the MISO function. An optimisation criterion was developed for this, which was minimised using a modified particle swarm (ME-D-PSO). In the work by T.-S. Wu *et al.* (2015), a control scheme based on Lyapunov stability was proposed to suppress the influence of external disturbances, eliminate fuzzy approximation errors, and ensure that the swaying motion of the tower crane system converges to the neighborhood of zero. In work by B.I. Mokin *et al.* (2020) a law of optimal control for one class of objects with time- and space-varying parameters (characteristics of the load movement process and the effect of longitudinal oscillations of the crane boom) was synthesised. An integral functional criterion with an integrand of the square of the deviations of the absolute angular velocity of the drive electric motor of the hoisting crane from its set function was used as the optimisation criterion. It showed that implementing the synthesised law of optimal control of the crane's electric drive achieves such characteristics of the load movement process that prevent the destruction of both the loads and their destination points, as the load movement process occurs without impact collisions. In work by I. Jebellat & I. Sharf (2023) two novel motion planning solutions for manipulator arms, by utilising the dynamic programming framework and collision-free paths time-dependent waypoints, were proposed. G.-H. Kim *et al.* (2021) offered a neural network-based robust antisway control method for the control problem of industrial cranes. To address the uncertainties in the system parameters and hydrodynamic forces, a neural network-based estimator was successfully designed to estimate the key functions in the control system, which avoids the high control gains required to guarantee robustness. I.O. Kadykalo (2021) developed a method of optimising the movement mode of the slewing mechanism of a jib crane during transitional processes, as well as a design of the drive mechanism that allows the implementation of optimal modes of motion the jib crane's slewing mechanism. Measurements were conducted using an encoder, an analog-to-digital converter, and a frequency converter. Also, the computer program "Optimal control of the jib crane turning mechanism" was developed to implement optimal modes of motion of the slewing mechanism. T.A. Le *et al.* (2013) presented control in the form of feedback for a rotating tower crane, combined with the movement of the trolley based on a sliding mode. The tower crane's developed motion controllers asymptotically stabilised the system's working state. S. Chwastek (2020) described a method for optimising crane mechanisms to reduce oscillations. The main advantage of the presented method is its simplicity and generality. This study underscores the need for a comprehensive approach to optimising multi-drive systems.

Researches E.Q. Hussein *et al.* (2020) presented the development of a nonlinear controller for the movement of a bridge crane. The procedure for designing the regulator with a sliding mode was detailed. Research showed that sliding mode control (SMC) performs better than a linear quadratic regulator (LQR) and requires less control force. Z. Sun *et al.* (2019) presented a sliding mode controller based on fuzzy logic (FSMC) to solve the problem of eliminating load oscillations that occur during the operation of bridge cranes. Computer simulation showed that FSMC based on the DE algorithm can effectively solve the problem of eliminating pendulum oscillations of the load. In work of S.-J. Kimmerle *et al.* (2018) a mathematical model of the problem of optimal duration movement of the "crane-trolley-load" system was presented. Based on the system's movement model, calculations were conducted to minimise the working cycle for moving load. S.M. Fasih *et al.* (2020) described the method of generating input data for effective control of rotation. The artificial neural network performs the function of suppressing fluctuations of the payload. It is used to develop a form of derivative zero vibration that can be updated according to the length of the rope. Y. Zuo *et al.* (2021) describe a method for quickly obtaining the voltage spectrum and calculating the crane's lifespan. An ANSYS finite element model is created based on static parameters and analyzed to determine the location of the dangerous point. In order to estimate the service life, the method of fracture mechanics is used in the work.

CONCLUSIONS

The problem of synthesising the optimal high-performance acceleration mode for the slewing mechanism of a tower crane has been established. The problem included a mathematical model of the system's motion, boundary conditions (defining the acceleration of the crane to the nominal rotation speed with the elimination of pendulum oscillations of the load), a performance criterion, constraints on the torque, and constraints on the angular velocity of the crane's rotation. Since it was a priori known that the solution to the problem of optimal performance was extreme values of the driving torque, an analytical solution to the system's equations of motion at constant driving torque was found. This allowed us to get derivation for the expressions for the final phase coordinates of the system's motion and the formation of a nonlinear criterion reflecting the degree of achievement of the final conditions and the duration of the system's movement. The problem of minimising the criterion was solved using the VCT-PSO algorithm with parameters $c_1 = c_2 = 1.19$, $RC = 5$, $w = 0.72$. An artificial neural network was formed to approximate the entire set of solutions to the problem. The neural network had 3 inputs (m, r, l), 3 outputs (t_1, t_2, t_3) and three hidden layers with five neurons each. The network training was conducted using the ADAM method. The root mean square and maximum errors in predicting the duration of the first, second, and third stages of the system's movement were determined (0.34%, 1.08%, 0.41%, respectively), and the maximums (1.44%, 7.34%, 1.63% respectively), indicating the high quality of data approximation. The indicators of the dynamic system's movement under optimal control were least affected by the mass of the load and most significantly by the length of the flexible suspension l. This was due to the strong influence of the value l to the period of the pendulum oscillations. Increasing the value l causes increasing in the duration of optimal control, and, consequently, a more intensive acceleration mode (both, in energy and dynamical senses). The developed approach to solving the problem of optimal control may be considered as a further development in the field of optimal (by duration) control. This contribution gave the general methodology to solve the optimal control problem. It was based on reducing the initial problem, multiple its solution, and training ANN to predict the switching moments of control, according to the parameters of the system to control. That is why the developed methodology may be applied to other dynamic systems (in particular, those corresponding to mechanisms of hoisting machinery: bridge and gantry cranes, lifts, robots, etc.). Future studies in this direction are connected with other parameters accounting. They are related to the mode of the system motion (constraints M_{max} and M_{min}, rated angular velocity ω), and initial values of the generalised coordinates (first of all and). In addition to that, ANN architecture and the volume of training data – are important issues to study, because of the fact, that they strongly influence the correctness of the switching moments t_1, t_2, t_3 prediction.

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CONFLICT OF INTEREST

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Синтез оптимального за швидкодією режиму розгону механізму повороту баштового крана

Анотація. Висока продуктивність вантажопідйомних кранів важлива для зведення будівель і споруд. Одним із ефективних способів цього досягти є усунення маятникових коливань вантажу на гнучкому підвісі баштових кранів. Тому метою статті було отримання загального розв'язку задачі оптимального за швидкодією керування протягом розгону механізму повороту крана. Представлена динамічна модель кранової системи, описані крайові умови, критерій оптимізації та введено обмеження на крутний момент. Виконано зведення вихідної задачі до задачі безумовної оптимізації нелінійної функції. Виконано багатократне розв'язування задачі, в якій маса вантажу m, довжина гнучкого підвісу l та виліт стріли r змінювались у певних межах. Для знаходження розв'язків використаний оптимізаційний алгоритм VCT-PSO. На основі отриманих розв'язків сформовано масив даних для навчання штучної нейронної мережі (навчальна вибірка) та масиву для перевірки її роботи (тестова вибірка). Для задачі прогнозування моментів перемикання керувань використано нейронну мережу прямого поширення із трьома входами (відповідають параметрам m, r, l), трьома виходами (відповідають моментам перемикання керувань) та трьома прихованими шарами з п'ятьма нейронами в кожному. Виконано тренування нейронної мережі за допомогою методу ADAM та визначено результати оцінок якості прогнозу моментів перемикання керувань, представлено графіки похибок оцінок. Встановлено високу якість апроксимації поверхні розв'язків задачі за допомогою штучної нейронної мережі. Проведено аналіз динаміки руху системи при оптимальному керуванні, які показав, що на показники руху системи найменший вплив має маса вантажу m, а найбільший – довжина гнучкого підвісу l

Ключові слова: баштовий кран; механізм повороту; оптимальне керування; маятникові коливання вантажу; штучна нейронна мережа