Machinery & Energetics
Vol. 16, No. 2. 2025

Journal homepage:
https://technicalscience.com.ua/en

UDC 631.312:514.18
DOI: 10.31548/machinery/2.2025.49

Andrii Nesvidomin
PhD in Technical Sciences, Associate Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0002-9227-4652

Serhii Pylypaka
Doctor of Technical Sciences, Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0002-1496-4615

Tetiana Volina*
PhD in Technical Sciences, Associate Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0001-8610-2208

Yurii Shtyka
PhD in Pedagogical Sciences, Associate Professor
Sumy State Pedagogical University named after A.S. Makarenko
40002, 87 Romenska Str., Sumy, Ukraine
https://orcid.org/0009-0002-5370-6739

Irina Rybenko
Senior Lecturer
Sumy National Agrarian University
40000, 160 Herasim Kondratiev Str., Sumy, Ukraine
https://orcid.org/0000-0001-7795-1689

Optimisation of a developable surface model passing through a helical curve with variable pitch

Abstract. Insufficient consideration of the developable helical surfaces in engineering practice complicates their manufacture and design, especially for variable pitch surfaces. The purpose of the article was to develop an algorithm for designing a helical surface of variable pitch and its mathematical implementation. For this purpose, the methods of differential geometry of curved lines and surfaces were used, as well as the MatLab software environment for computing, data analysis, visualisation and development of algorithms for constructing surfaces based on the results obtained. The basis for the surface construction was a spiral line of variable pitch, which can be specified by various dependencies. The task was to draw a set of rectilinear surface components through this spiral line with a vertical axis, provided that it was a developable surface. An additional condition was that these lines must be inclined at a constant angle to the horizontal

---

Article's History: Received: 30.11.2024; Revised: 17.03.2025; Accepted: 22.05.2025.

Suggested Citation:
Nesvidomin, A., Pylypaka, S., Volina, T., Shtyka, Yu., & Rybenko, I. (2025). Optimisation of a developable surface model passing through a helical curve with variable pitch. *Machinery & Energetics,* 16(2), 49-57. doi: 10.31548/machinery/2.2025.49.

*Corresponding author

plane, i.e. the receptive surface must be a surface of equal inclination of the lines. Usually, the unfolding surface is defined by a spatial curve – an edge of inverse. The set of straight-line tangents to the back edge forms the developable surface. However, in practical problems, it is important to ensure that the scan surface passes through a given curve, for example, a helical line. It has been established that a set of reamer surfaces of the same slope of the constituent parts with different specified angles can be drawn through a given helical line. It was proved that it is easy to obtain a surface compartment bounded by two coaxial cylinders, one of which has a given helical line. The results of the study can be used to improve the technology of manufacturing screws in agriculture, food, mining and construction industries

Keywords: rectilinear constituent parts; guide cosines; slope surface; envelope curve; screw

INTRODUCTION

The study of the optimisation of the developable surface model is necessary to improve the technology of manufacturing augers in agriculture, food, mining and construction industries, as it allows for increased accuracy of geometric parameters, reduced material waste and improved performance of the final product. In agriculture, augers are used to transport grain, feed and fertilisers, where it is important to distribute the material evenly and minimise damage. In the food industry, they are used in the production of flour, sugar, coffee and other bulk products, where the precise shape of the auger ensures efficient movement without sticking and loss. The mining industry requires high wear resistance and durability of screws for transporting abrasive materials, while the construction industry uses them in concrete mixers, drilling rigs and bulk material feeding systems. Optimising the shape of the developable surface improves design solutions, simplifies production and reduces energy costs for the operation of screw mechanisms, making them more efficient and cost-effective

Helical surfaces play an important role in various fields of technology, which is confirmed by numerous scientific studies. M. Mushtruk *et al.* (2020) investigated the use of helical surfaces in twin-screw extruders, in particular for the process of extruding oil from pre-ground raw materials. Their work can improve the efficiency of the process and the quality of the final product. S. Pylypaka *et al.* (2024a) proposed the use of a helical surface for the design of a screw knife of a grinding drum, which is relevant for increasing the efficiency of grinding materials in industrial and agricultural machines. S. Liu *et al.* (2023) optimised the design of a screw pair in a hydraulic rotary actuator, which contributes to improving the performance and efficiency of hydraulic systems.

Researchers A. Rucins *et al.* (2024) focused on the study of the energy parameters of a screw conveyor with a paddle impeller used to transport agricultural materials. The study is aimed at reducing energy consumption and optimising the operation of transport mechanisms. Similarly, V. Bulgakov *et al.* (2024) considered the traction resistance of harrows with screw implements, which is important for the design of modern agricultural machinery.

Scientist L. Huran (2024) studied the computer modelling of rotating cutting tools with helical teeth, which can improve the manufacturing accuracy and performance of such tools in mechanical engineering. In their work, J. Zhao *et al.* (2024) investigated the evolution of tooth surface morphology in the process of mixed wear during sliding contact under mixed lubrication. They found patterns of changes in the microrelief of the tooth surface, which has a significant impact on their wear resistance and tribological characteristics. This study contributes to the improvement of methods for predicting the durability of helical gears and the selection of materials and coatings to increase their durability. J. Chen *et al.* (2024) focused on general meshing modelling and analysis of the dynamic characteristics of a helical gear with tooth profile deviations. Their work allowed them to determine the impact of different types of defects on gear dynamics, which is important for improving the design of gears and increasing their operational reliability.

Special attention is paid to the optimisation of screw turbines. J.C.P. Ortiz *et al.* (2024) presented a methodology for optimising the design of a throat screw turbine for hydrokinetic applications using experimental studies and response surface methodology. This research has increased the efficiency of energy production from water flow, which is a promising area in renewable energy.

As a result of reviewing the sources on the chosen topic, it was found that the analysis of helical surfaces for their internal structure in terms of differential geometry is not given due attention. Meanwhile, the surface can be developable or non-developable, which should be taken into account in the technology of manufacturing turns. A developable surface allows to find its exact ream, which is formed into a finished product by simply bending it without significant plastic deformations. That is why the aim of the study was to find helical reamers that pass through a given helical line

MATERIALS AND METHODS

The study used the symbolic mathematics of the Mathematica software product, as well as the MatLab software environment to visualise the results. The task of constructing the surface was to draw a set of rectilinear lines through a given spiral line with a vertical axis so that the resulting surface was developable. In addition, a condition was imposed on the surface, according to which it had to be a surface of equal slope of the constituent lines. The method of constructing the surface was based on a well-known method of creating surfaces of equal slope of the constituent elements. In practice, such surfaces can be observed on the slopes of motorways that have a curved axis and go uphill. Bulk soil has a constant angle of inclination to the horizontal plane, the value of which depends on its properties. The edge of the road was assumed to be a spatial curve 1 on a cylinder of radius R (Fig. 1a). At certain points on the curve, sand was poured out, resulting in cones of different heights z with the same angle β of natural slope (Fig. 1b). If such points on the curve are located with a high density, two scanning surfaces of the same slope of the constituent surfaces were formed. Each of the tangents passed through the top of the cone and intersected curves 2 and 3 in the horizontal plane from which the ascent began. These curves were enveloping for a set of circles – the bases of the cones, with a straight line passing through the top of the cone and the point of contact of its base with the enveloping curve. The object under study was a surface located outside the cylinder (in Fig. 1a, the individual generating surfaces are marked with thickened segments).

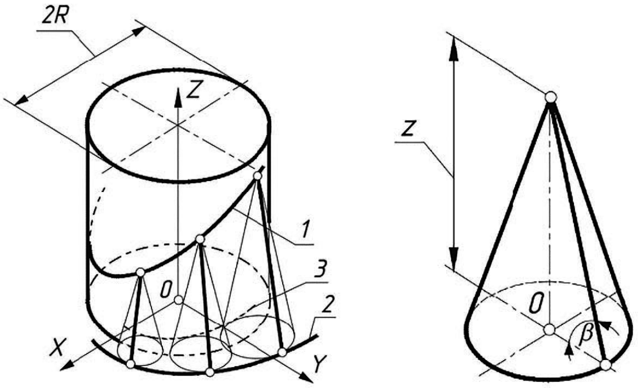

Figure 1. Graphical illustrations of the formation of a scanning surface of the same slope of the derivatives

Note: a) a scheme of surface formation by constructing cones with vertices on a given cylindrical curve; b) a single cone whose height z depends on the location of its vertex on a given curve: OXYZ – spatial coordinate system; 2R – diameter of the cylinder; z – height of the cone; β – angle of inclination of the cone's constituent parts to its base (angle of natural slope); 1 – a given curve, 2, 3 – envelope curves of circles – bases of cones

Source: developed by the authors

The radius of the base of the cone r_o was variable and depended on its height z, i.e. its value was determined from the expression $r_o = z\text{-}\cdot ctg\beta$. The cylindrical curve on which the cone vertices are located was given by the parametric equations:

$$x = R\cos\alpha;\ y = R\sin\alpha;\ z = z(\alpha), \tag{1}$$

where R is the radius of the cylinder; α is the angle of rotation of the curve point around the cylinder axis when the cone vertex moves along it.

The centres of all the circles – the bases of the cones – were located on a circle of radius R, the horizontal projection of the cylinder. In this case, the one-parameter set of circles – the bases of the cones – is described by the implicit equation:

$$(x - R\cos\alpha)^2 + (y - R\sin\alpha)^2 = z^2\mathrm{ctg}^2\,\beta. \tag{2}$$

Differentiating equation (2) by the variable α yielded the result:

$$2R\sin\alpha\,(x - R\cos\alpha) - 2R\cos\alpha\,(y - R\sin\alpha) = 2zz'\mathrm{ctg}^2\,\beta. \tag{3}$$

The system of two equations (2) and (3) was solved with respect to x and y. These were the coordinates of the points of the envelope curve, which were denoted as x_o and y_o:

$$x_o = R\cos\alpha + \frac{z\mathrm{ctg}\beta}{R}\left[z'\mathrm{ctg}\beta\sin\alpha \pm \sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\cos\alpha\right];$$

$$y_o = R\sin\alpha - \frac{z\mathrm{ctg}\beta}{R}\left[z'\mathrm{ctg}\beta\cos\alpha \mp \sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\sin\alpha\right]. \tag{4}$$

If a certain dependence $z = z(\alpha)$ was given, then parametric equations (4) described two curves, one of which corresponded to the upper sign before the square root, and the other to the lower sign. These were the envelope curves, which are marked with numbers *2* and *3* in Figure 1. Since these curves were in the horizontal plane, $z_o = 0$. Thus, having the coordinates of the current point of the curve (1) with the given dependence $z = z(\alpha)$ and the coordinates of the corresponding point of the envelope (4), it is possible to construct a rectilinear surface by connecting these points with a segment.

RESULTS AND DISCUSSION

A diagram of a developable surface is shown in Figure 2. It shows several rectilinear surface elements. A section of the surface between uniaxial cylinders of radii R and r is considered. In this case, the straight line of the surface must be limited to the segment between these cylinders (for example, segment AB, Fig. 2). To construct a rectilinear surface that extends from a point on a given cylindrical curve, you need to know the unit directional vector. To determine it, you can use the well-known expression for a line passing through two points in space:

$$\frac{x_n - x_o}{x - x_o} = \frac{y_n - y_o}{y - y_o} = \frac{z_n - z_o}{z - z_o} = u, \tag{5}$$

where $x_{(n)}$, $y_{(n)}$, z_n are the coordinates of the current point of the line product; u is a parameter proportional to the length of this line product.

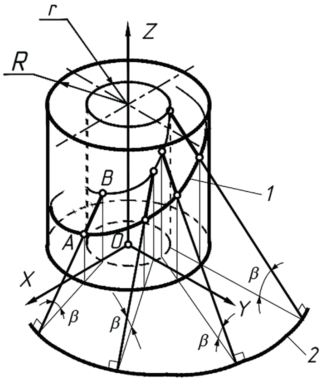

Figure 2. Schematic of the formation of a developable surface with a compartment bounded by two coaxial cylinders

Note: *R* and *r* – radius of uniaxial cylinders (outer and inner, respectively); *OXYZ* – spatial coordinate system; *AB* – segment of a straight line of the generating surface bounded by cylinders; *β* – angle of inclination of the generating surface to the horizontal plane; *1* – specified curve; *2* – envelope curve

Source: developed by the authors on the basis of research

The expressions in the denominator of equation (5) are the projections *l, m, n* of the directional vector of the rectilinear generating surface. After substitution of the corresponding equations (1) and (4) and after simplifications:

$$\begin{gathered} l = -\frac{\text{ctg}\beta}{R}\left(z^{'}\text{ctg}\beta \sin\alpha \pm \sqrt{R^2 - z^{'2}\text{ctg}^2\beta} \cos\alpha\right); \\ m = \frac{\text{ctg}\beta}{R}\left(z^{'}\text{ctg}\beta \cos\alpha \mp \sqrt{R^2 - z^{'2}\text{ctg}^2\beta} \sin\alpha\right); \\ n = 1. \end{gathered} \tag{6}$$

The vector (6) is not unit. It is normalised to unit by dividing its components by its modulus:

$$\begin{gathered} l = -\frac{\cos\beta}{R}\left(z^{'}\text{ctg}\beta \sin\alpha \pm \sqrt{R^2 - z^{'2}\text{ctg}^2\beta} \cos\alpha\right); \\ m = \frac{\cos}{R}\left(z^{'}\text{ctg}\beta \cos\alpha \mp \sqrt{R^2 - z^{'2}\text{ctg}^2\beta} \sin\alpha\right); \\ n = \sin\beta. \end{gathered} \tag{7}$$

In the projections of vector (7), the square root can be signed either upper or lower. The test showed that for the problem under study, the lower sign is required. It corresponds to the surface products that extend from the envelope *3* in Figure 1a. With this equation in mind, the surfaces in which all the rectilinear products pass through the given curve (1) parallel to the directional unit vector (7) are written:

$$\begin{gathered} X = R\cos\alpha - u\frac{\cos\beta}{R}\left(z^{'}\text{ctg}\beta \sin\alpha - \sqrt{R^2 - z^{'2}\text{ctg}^2\beta} \cos\alpha\right); \\ Y = R\sin\alpha + u\frac{\cos}{R}\left(z^{'}\text{ctg}\beta \cos\alpha + \sqrt{R^2 - z^{'2}\text{ctg}^2\beta} \sin\alpha\right); \\ Z = z(\alpha) + u\sin\beta, \end{gathered} \tag{8}$$

where u is the second independent variable of the surface, which is the length of the rectilinear product and whose reference starts from the given cylindrical curve.

All the rectilinear generating surfaces of (8) are tangent to the spatial curve – the inverse edge. Two infinitely close generating surfaces intersect at a point on the inverse edge, and it is formed by a set of such points. First, you need to find the horizontal projection of the back edge. To do this, the variable u must be eliminated from the first two equations (8). The result of the simplifications is as follows:

$$\begin{gathered} (X\sin\alpha - Y\cos\alpha)\sqrt{R^2 - z^{'2}\text{ctg}^2\beta} - \\ -z^{'}\text{ctg}\beta(X\cos\alpha + Y\sin\alpha - R) = 0. \end{gathered} \tag{9}$$

Equation (9) describes a one-parameter set of rectilinear surface components in the projection onto a horizontal plane. The position of an individual line depends on the value of the parameter α. Differentiation of (9) with respect to the variable α gives another equation, which includes the notation X and Y. As a result, there are two equations in which X and Y are unknown dependencies. The resulting system needs to be solved with respect to these dependencies. These will be the parametric equations of the horizontal projection of the back edge. In this case, the uppercase letters X and Y, which were related to the surface, are replaced with lowercase letters with the index "p", since they no longer refer to the surface, but to the curve – the back edge. The intermediate results of the described operations, which were performed using the symbolic mathematics of the Mathematica software product, are not shown because of their cumbersome appearance. The finished result (parametric equations of the back edge) has the form:

$$\begin{gathered} x_p = \frac{\left(R^2 z^{''} + z^{'2}\text{ctg}\beta\sqrt{R^2 - z^{'2}\text{ctg}^2\beta}\right)\cos\alpha + z^{'}\left(R^2 - z^{'2}\text{ctg}^2\beta\right)\sin\alpha}{R\,\text{tg}\beta\left(z^{''}\text{ctg}\beta + \sqrt{R^2 - z^{'2}\text{ctg}^2\beta}\right)}; \\ y_p = \frac{\left(R^2 z^{''} + z^{'2}\text{ctg}\beta\sqrt{R^2 - z^{'2}\text{ctg}^2\beta}\right)\sin\alpha - z^{'}\left(R^2 - z^{'2}\text{ctg}^2\beta\right)\cos\alpha}{R\,\text{tg}\beta\left(z^{''}\text{ctg}\beta + \sqrt{R^2 - z^{'2}\text{ctg}^2\beta}\right)}; \\ z_p = z + \frac{z^{'2} - R^2\text{tg}^2\beta}{z^{''} + \text{tg}\beta\sqrt{R^2 - z^{'2}\text{ctg}^2\beta}}. \end{gathered} \tag{10}$$

The last equation in (10) $z_p = {}_{z(p)}(\alpha)$ can be obtained separately after finding the first two. To do this, the $x_{(p)}$ of the back edge (10) and the X of the surface (8) are equated. From the resulting equation, an expression for u is found, which is then substituted into the last equation (8). You can equate u_p and Y. The result will be the same.

The visualisation of the back edge and the rectilinear surface elements was performed using the MatLab software. Any relationship $z = z(\alpha)$ can be specified and a line on the cylinder can be obtained using equations (1). However, this does not mean that it is possible to construct a scan surface along its entire length. This is demonstrated by an example. Let $z = e^{(0{,}2\alpha)}$, $z' = 0.2e^{(0{,}2\alpha)}$, $z'' = 0.04e^{(0{,}2\alpha)}$. Figure 3 shows the surface section according to equations (8) when the parameter α varies within the range $\alpha = 0 \ldots 3\pi$: in Figure 3a – in axonometry, in Figure 3b – horizontal projection of the surface section. For wider limits of change in the parameter α, the construction of the surface becomes impossible. This is due to the value of the root expression in Equations (8) when it becomes negative. Its value is also affected by the value of the angle α. The length of the straight line is 2 linear units ($u = 0 \ldots 2$). According to equations (10), the back edge is constructed with a thickened line. For some straight-line generators, it is shown that after their continuation, they touch the back edge.

The horizontal projection (Fig. 3b) shows that the outer edge is not a cylindrical line. To eliminate this drawback, a transition from the parameter u (the length of the rectilinear product) to the parameter ρ, which represents the distance from the axis to the surface points, is required. This distance is found by a well-known formula using the first two equations (8):

$$\rho = \sqrt{X^2 + Y^2} = \sqrt{R^2 + u\cos\beta\left(u\cos\beta + 2\sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\right)}. \tag{11}$$

After substituting the expression u, found from equation (11), into equation (8), they take the final form:

$$X = R\cos\alpha + \frac{1}{R}\left(\sqrt{\rho^2 - z'^2\mathrm{ctg}^2\beta} - \sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\right)\left(\sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\cos\alpha - z'\mathrm{ctg}\beta\sin\alpha\right);$$

$$Y = R\sin\alpha + \frac{1}{R}\left(\sqrt{\rho^2 - z'^2\mathrm{ctg}^2\beta} - \sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\right)\left(\sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\sin\alpha + z'\mathrm{ctg}\beta\cos\alpha\right);$$

$$Z = z(\alpha) + \left(\sqrt{\rho^2 - z'^2\mathrm{ctg}^2\beta} - \sqrt{R^2 - z'^2\mathrm{ctg}^2\beta}\right)\mathrm{tg}\beta. \tag{12}$$

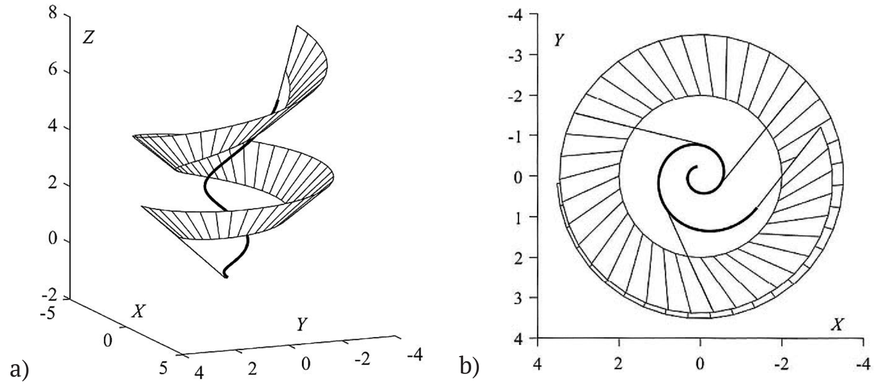

Figure 3. Surface section and the corresponding back edge constructed according to equations (8) and (10) at $R = 2$ and $\beta = 40°$

Note: a) axonometric image; b) horizontal projection
Source: developed by the authors on the basis of research

The obtained equations (12) make it possible to consider the design of a variable pitch sweeping helical surface. Such a surface can be used as an auxiliary surface in forage harvesters. Two rotating augers are used to direct the plant mass into the neck, moving the mass towards the middle, i.e. two streams of plant mass move towards each other. The main screw, which moves the crop, is reinforced with welded braces, but it is more reliable to use a retaining screw, which can be made from a reamer. If the pitch of the screw surface is constant, the speed of the oncoming flows of plant mass is also constant. As these flows approach the centre, their speed should decrease, and in the centre, they should be zero. This can be achieved by using a variable pitch helical surface. In order for the mass velocity to decrease according to a linear law, the relationship $z = z(\alpha)$ must have the following form:

$$z = a\alpha - b\alpha^2, \tag{13}$$

where a and b are constant values. In the special case when $b = 0$, the surface will have a constant pitch.

Based on the average dimensions of the combine's working width and the inner (shaft radius) and outer edges of the helical surface ($\rho = 125$ mm and $\rho = 250$ mm), the values of the constants a and b in equation (13) were found to be

$a=100$ and $b=1.5$. For these data at $\beta=40°$, a scanning helical surface was constructed by changing the parameter ρ in the range of $\rho=125...250$ and changing the angle α in the range of $\alpha=0...8\pi$. The result of the construction is shown in Figure 4a,b. In Figure 4a, this surface is built on a shaft with a radius *of* $r=125$ *mm* with a horizontal axis, and a working surface in the form of a helical conoid is also completed.

The common helical line (the outer edge of the main and auxiliary surfaces) in Figure 4c is shown as a thickened line. The research conducted is important from the point of view of the practical application of the results obtained. As noted at the beginning of the article, in engineering practice, insufficient attention is paid to the geometry of helical surfaces. They can be linear and non-linear. Linear helical surfaces (helicoids) are the most widely used in practice. In turn, they can be developable and non-developable. The former have a simple analytical description, but require special equipment for their manufacture. There is no exact scan for them, and an approximate one requires plastic deformation and metal redistribution. For developable surfaces, an exact develop can be constructed based on the constancy of the area, line lengths and angles between them, but this requires a complex mathematical description using differential geometry. The production of helical surfaces from a flat workpiece is accomplished by simply bending it, similar to folding a sheet of paper into a cylindrical surface. In this case, plastic deformation is minimal and is due to the thickness of the sheet material.

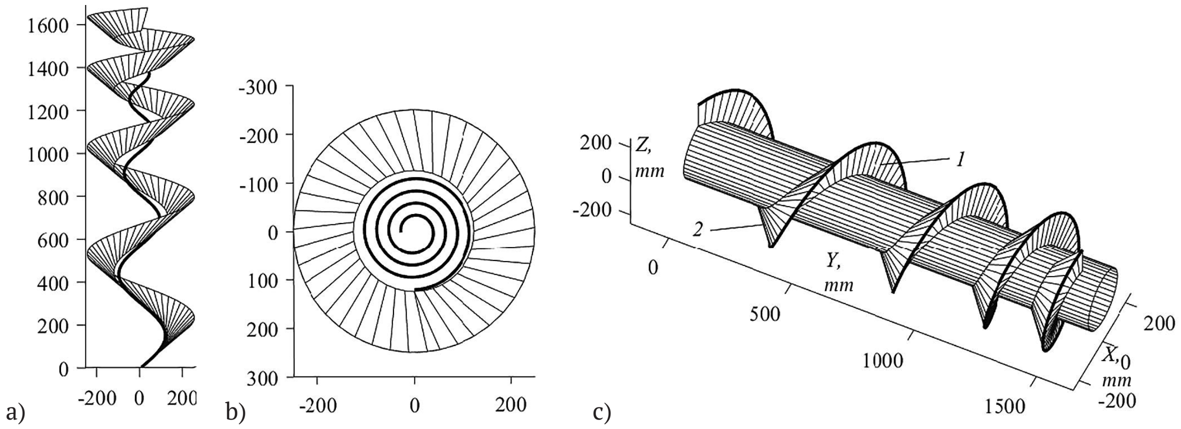

Figure 4. Variable pitch helical developable surface and its application

Note: a) frontal projection of the surface and its reverse edge; b) horizontal projection of the surface and its reverse edge; c) use of a developable screw surface: 1 – working non-developable surface (helical conoid), 2 – auxiliary (backing) surface (variable pitch developable surface)

Source: developed by the authors on the basis of research

All the developable surfaces have a reverse edge to which the rectilinear generating surfaces are tangent (for cones it degenerates into a point, and for cylinders – into an infinitely distant point). All the generating surfaces are located on one side of it (Fig. 3b, Fig. 4b). For this reason, for a helical surface, it must be inside the cylinder on which the given line is located. Therefore, along the entire length of this line, you can build a developable surface with a given angle of inclination of the generative elements β. As soon as the back edge enters the cylindrical surface, further surface construction becomes impossible. In this case, the root expression in the surface equations (8) becomes negative. The root expression also includes the angle β, so it also affects the surface construction process. For the surface (Fig. 4) on a certain section of a given helical line of variable pitch, it is close to the minimum and is $\beta=40°$. The minimum value of the angle β can be determined from the root expression in equations (8) by equating it to zero. From there: $\beta=Arctg(z'/R)$, which for the case under consideration is 38.7°. At the lowest point of the surface (Fig. 4a, b), the back edge extends to the surface of the cylinder, i.e. the straight line becomes tangent to it. Further design of the developable surface below this point becomes impossible, as the law of its formation is violated. It is possible to increase the angle β, so that a certain part of the surface can be constructed downwards, but this increases the distance between the seam of the main and support surfaces on the shaft, which leads to a decrease in the useful volume between turns.

In the work of S. Pylypaka *et al.* (2024b) proposed a helical sweeping surface of constant pitch to be used as a working body of a helical knife of an agricultural implement. The given helical line is the cutting edge. Straight lines pass through it, the inclination of which is chosen to reduce the resistance of the working body to entering the material. The main difference between the surface design in this and the proposed work is that the surface of the working body has a constant pitch and the back edge is also a helical line of constant pitch. This is a special case that can be obtained from dependence (13) of our work when $b=0$. In this case, $z'=a$, $z''=0$ and equation (10) will describe a helical line of constant pitch.

E. Güler & Y. Yayli (2023) and E. Güler & N.C. Turgay (2024) investigated the geometric isometries of helical surfaces in four-dimensional and five-dimensional Euclidean space. They analysed the properties and symmetry of such surfaces using mathematical methods of their description and classification. However, the authors did not focus on the practical application or design of such surfaces, unlike the current study.

Numerous studies have been conducted on the design of non-rollable surfaces. A. Bankova *et al.* (2024) proposed a method for develop of non-developable surfaces from sheet material, which can be used in the production of complex structures. L. Zawallich & R. Pajarola (2024) considered a mesh approximation of non-developable surfaces, which allows simplifying their geometric model and facilitating the unwrapping process. These studies highlighted the importance of accurate methods for designing such surfaces, which is critical for manufacturing processes.

W. Zhang *et al.* (2024) investigated the process of manufacturing helical surfaces by milling. They focused on the effect of the machining process on energy consumption and the quality of the machined surface. The disadvantage of this approach was the high energy consumption, which requires further process optimisation. In addition, an alternative method of manufacturing screw surfaces, such as bending, was considered by A. Kumar *et al*. (2024). The authors noted that the develop of non-developable surfaces can only be approximate, and their bending into the desired shape significantly increases the energy intensity of the process. Thus, these studies have shown that further improvements in the manufacturing of screw surfaces are needed to reduce costs and increase efficiency.

Scientists V. Tarelnyk *et al.* (2019a; 2019b) studied surface plastic deformations and their impact on the microgeometry, structure, wear resistance and efficiency of coatings. The studies showed that the use of combined surface treatment methods improves the performance of parts. However, the authors did not take into account the impact of surface geometry on such indicators at the design stage, which opens up opportunities for further research in this area.

The question of the construction of developable surfaces was addressed by A.S.M. Kamarudzaman & M.Y. Misro (2024), where the authors studied the develop of the quintic trigonometric Bezier surface. T.G. Nelson *et al.* (2019) proposed approaches to the design of developable surfaces and detailed the areas of their possible application, in particular in robotics, the creation of compact devices and adaptive structures. F. Tash & R. Ziatdinov (2023) considered the developable linear surfaces formed by using the curvature axis of the curve. They investigated the geometric properties of such surfaces and proposed methods for their construction.

In general, a considerable amount of research has been aimed at studying the properties and methods of constructing helical and developable surfaces. However, many works focus only on theoretical aspects, without considering practical applications, unlike the current study.

CONCLUSIONS

A method for constructing a developable surface of the same slope of the constituent elements that passes through a given cylindrical line is developed. The task of the work was to find a set of rectilinear surface components that would form a developable surface. In addition, all of the generating surfaces must have the same angle of inclination to the horizontal plane. By setting the angles of inclination to different constant values, a set of reamer surfaces passing through a given cylindrical line can be obtained. To solve this problem, a geometric model of the formation of a developable surface was proposed. It consisted of constructing a one-parameter set of cones with a given angle of inclination of their generators to the horizontal plane and a vertex on a given cylindrical curve. The developable surface of such a set of cones is the developable surface. The mathematical implementation of the proposed model consisted in finding the curve of the section of the develop by the horizontal plane. In this plane, a one-parameter set of circles is formed – the sections of cones. The radius of each circle depends on the distance from the horizontal plane to the vertex of the cone, which is located on a given cylindrical line. Since this height is known, the radius of the bases of all cones are known. This circumstance made it possible to find the circumscribing lines of the set of circles in the horizontal plane with their points of contact. The intersection of the point of contact of a particular cone with its vertex determines the rectilinear product of the surface. The parametric equations of the surface and its back edge are obtained. The equations allow us to obtain a surface compartment located between two coaxial cylinders. This is made possible by the fact that one independent variable of the surface equations is the distance from its axis to points on it. At a constant value of this parameter, a bounding cylindrical line of a given radius is formed on the surface. A surface cannot be constructed along the entire length of a given cylindrical line, since a given angle of inclination of its rectilinear components in a certain section of the curve will contradict the law of its formation. Mathematically, this limitation is due to the value of the root expression included in the surface equations, since this expression cannot be negative. The direction of further research should be the construction of a scan of the surface compartment bounded by two coaxial cylinders. For helical surfaces with a constant pitch, such compartments are flat figures in the form of a ring, the bounding radius of which are determined by the formulas. If the pitch is variable, the bounding curves are spiral curves, which require special algorithms to be developed.

ACKNOWLEDGEMENTS

The authors of the article express their gratitude to the soldiers of the Armed Forces of Ukraine, whose courage makes it possible for Ukrainian scientists to do science.

FUNDING

None.

CONFLICT OF INTEREST

None.

REFERENCES

[1] Bankova, A., Tenev, S., Atanasov, A., Nikolov, P., Mehmedov, I., & Neveda, N. (2024). Construction of the intersection lines of a sheet material transition and design of the unfolding of its surface. In *5th international conference on communications, information, electronic and energy systems (CIEES)* (pp. 1-5). Veliko Tarnovo: IEEE. doi: 10.1109/CIEES62939.2024.10811193.

[2] Bulgakov, V., Rucins, A., Holovach, I., Trokhaniak, O., Klendii, M., Popa, L., & Kutsenko, A. (2024). Theoretical study of traction resistance of harrows with helical working bodies. *INMATEH - Agricultural Engineering*, 74(3), 380-387. doi: 10.35633/inmateh-74-33.

[3] Chen, J., Zhu, R., Chen, W., Li, M., & Yin, X. (2024). General meshing modelling and dynamic characteristics analysis of helical gear pair with tooth surface deviation. *Iranian Journal of Science and Technology Transactions of Mechanical Engineering*, 48(1), 1623-1641. doi: 10.1007/s40997-024-00751-4.

[4] Güler, E., & Turgay, N.C. (2024). Analysing geometric isometries of helical surfaces in five-dimensional Euclidean space. *Filomat*, 38(23), 8121-8129. doi: 10.2298/FIL2423121G.

[5] Güler, E., & Yayli, Y. (2023). Local isometry of the generalised helicoidal surfaces family in 4-space. *Malaya Journal of Mathematik*, 11(2), 210-218. doi: 10.26637/mjm1102/009.

[6] Huran, L. (2024). The forming theory and computer simulation of the rotary cutting tools with helical teeth and complex surfaces. *Computer and Information Science*, 1(4), 158-158. doi: 10.5539/cis.v1n4p158.

[7] Kamarudzaman, A.S.M., & Misro, M.Y. (2024). Developability comparison of enveloping developable quintic trigonometric Bézier surfaces. *AIP Conference Proceedings*, 3150, article number 030005. doi: 10.1063/5.0228304.

[8] Kumar, A., Chandan, A., & Mahato, A. (2024). Multi-scale surface folding in metal cutting. *Journal of Manufacturing Processes*, 120, 628-640. doi: 10.1016/j.jmapro.2024.04.070.

[9] Liu, S., Li, B., Gan, R., & Xu, Y. (2023). Surrogate-based optimisation design for surface texture of helical pair in helical hydraulic rotary actuator. *Scientific Reports*, 13(1), article number 20259. doi: 10.1038/s41598-023-47509-7.

[10] Mushtruk, M., Gudzenko, M., Palamarchuk, I., Vasyliv, V., Slobodyanyuk, N., Kuts, A., Nychyk, O., Salavor, O., & Bober, A. (2020). Mathematical modelling of the oil extraction process with pre-grinding of raw materials in a twin-screw extruder. *Potravinarstvo Slovak Journal of Food Sciences*, 14, 937-944. doi: 10.5219/1436.

[11] Nelson, T.G., Zimmerman, T.K., Magleby, S.P., Lang, R.J., & Howell, L.L. (2019). Developable mechanisms on developable surfaces. *Science Robotics*, 4(27), article number eaau5171. doi:10.1126/scirobotics.aau5171.

[12] Ortiz, J.C.P., Rubio-Clemente, A., & Chica, E. (2024). Optimisation of a Gorlov helical turbine for hydrokinetic application using the response surface methodology and experimental tests. *Energies*, 17(22), article number 5747. doi: 10.3390/en17225747.

[13] Pylypaka, S., Hropost, V., Nesvidomin, V., Volina, T., Kalenyk, M., Volokha, M., Zalevska, O., Shuliak, I., Dieniezhnikov, S., & Motsak, S. (2024a). Designing a helical knife for a shredding drum using a sweep surface. *Eastern-European Journal of Enterprise Technologies*, 4(1(130)), 37-44. doi: 10.15587/1729-4061.2024.308195.

[14] Pylypaka, S., Khropost, V., Volina, T., Kresan, T., Hryshchenko, I., Babka, V., Rybenko, I., & Rebriy, A. (2024b). Features of bending of curvilinear absolutely elastic bars. *Scientific Reports of the National University of Life and Environmental Sciences of Ukraine*, 20(3). doi: 10.31548/dopovidi.3(109).2024.019.

[15] Rucins, A., Bulgakov, V., Holovach, I., Trokhaniak, O., Klendii, M., Popa, L., & Yaremenko, V. (2024). Research on power parameters of a screw conveyor with bladed operating body for transporting agricultural materials. *INMATEH - Agricultural Engineering*, 74(3), 428-435. doi: 10.35633/inmateh-74-38.

[16] Tarelnyk, V.B., Gaponova, O.P., Konoplianchenko, Ye.V., Martsynkovskyy, V.S., Tarelnyk, N.V., & Vasylenko, O.O. (2019a). Improvement of quality of the surface electroerosive alloyed layers by the combined coatings and the surface plastic deformation. III. The influence of the main technological parameters on the microgeometry, structure and properties of electrolytic erosion coatings. *Metallofizika i Noveishie Tekhnologii*, 41(3), 313-335. doi: 10.15407/mfint.41.03.0313.

[17] Tarelnyk, V.B., Gaponova, O.P., Konoplianchenko, Ye.V., Martsynkovskyy, V.S., Tarelnyk, N.V., & Vasylenko, O.O. (2019b). Improvement of quality of the surface electroerosive alloyed layers by the combined coatings and the surface plastic deformation. II. The analysis of a stressedly-deformed state of surface layer after a surface plastic deformation of electroerosive coatings. *Metallofizika i Noveishie Tekhnologii*, 41(2), 173-192. doi: 10.15407/mfint.41.02.0173.

[18] Tash, F., & Ziatdinov, R. (2023). Developable ruled surfaces generated by the curvature axis of a curve. *Axioms*, 12(12), article number 1090. doi: 10.3390/axioms12121090.

[19] Zawallich, L., & Pajarola, R. (2024). Unfolding via mesh approximation using surface flows. *Computer Graphics Forum*, 43(2), article number e15031. doi: 10.1111/cgf.15031.

[20] Zhang, W., Sun, X., Yang, H., Liu, Y., & Dong, Zh. (2024). A process parameters decision approach considering spindle vibration in helical surface milling for minimising energy consumption and surface roughness value. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 46, article number 675. doi: 10.1007/s40430-024-05242-w.

[21] Zhao, J., Ma, Ch., Li, Zh., Yu, X., & Sheng, W. (2024). Evolution of tooth surface morphology and tribological properties of helical gears during mixed lubrication sliding wear. *Surface Topography: Metrology and Properties*, 12(3), article number 035037. doi: 10.1088/2051-672X/ad76c3.

Андрій Несвідомін
Кандидат технічних наук, доцент
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0002-9227-4652
Сергій Пилипака
Доктор технічних наук, професор
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0002-1496-4615
Тетяна Воліна
Кандидат технічних наук, доцент
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0001-8610-2208
Юрій Штика
Кандидат педагогічних наук, доцент
Сумський державний педагогічний університет імені А.С. Макаренка
40002, вул. Роменська, 87, м. Суми, Україна
https://orcid.org/0009-0002-5370-6739
Ірина Рибенко
Старший викладач
Сумський національний аграрний університет
40000, вул. Герасима Кондратьєва, 160, м. Суми, Україна
https://orcid.org/0000-0001-7795-1689

Оптимізація моделі розгортної поверхні, яка проходить через гвинтову лінію змінного кроку

Анотація. Недостатнє врахування розгортності гвинтових поверхонь в інженерній практиці ускладнює їх виготовлення та конструювання, особливо для поверхонь змінного кроку. Метою статті була розробка алгоритму конструювання гвинтової поверхні змінного кроку та його математична реалізація. З цією метою були задіяні методи диференціальної геометрії кривих ліній і поверхонь, а також середовище програмного продукту для обчислень, аналізу даних, візуалізації та розробки алгоритмів MatLab для побудови поверхонь за отриманими результатами. Основою для побудови поверхні взято гвинтову лінію змінного кроку, який може бути заданий різними залежностями. Задача полягала в тому, щоб через цю гвинтову лінію із вертикальною віссю провести множину прямолінійних твірних поверхні за умови, щоб вона була розгортною. Додаткова умова – ці твірні мають бути нахилені під сталим кутом до горизонтальної площини, тобто розгортна поверхня має бути поверхнею однакового нахилу твірних. Зазвичай розгортну поверхню задають просторовою кривою – ребром звороту. Множина прямолінійних дотичних до ребра звороту утворює розгортну поверхню. Однак у практичних задачах важливо забезпечити проходження розгортної поверхні через задану криву, наприклад, гвинтову лінію. Встановлено, що через задану гвинтову лінію можна провести множину розгортних поверхонь однакового нахилу твірних із різними заданими кутами. Доведено, легко отримати відсік поверхні, обмежений двома співвісними циліндрами, на одному із яких розташована задана гвинтова лінія. Результати дослідження можуть бути використані для покращення технологій виготовлення шнеків у сільському господарстві, харчовій, гірничодобувній та будівельній галузях

Ключові слова: прямолінійні твірні; напрямні косинуси; поверхня укосу; обвідна крива; шнек