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Mathematical modelling of oscillations of a machine for cutting tops of root crops

Abstract. High-quality cutting of root crop tops at the root ensures both a high yield and determines the condition of the root crops before they are extracted from the soil. Therefore, the search for conditions that will ensure this is an important and urgent problem for the production of these crops. The purpose of this research was to determine, through an analytical study, the optimal parameters of a new design of a machine for harvesting root crop tops, which is mounted in front of an aggregating tractor and performs oscillatory movements in the longitudinal-vertical plane. Methods related to the modelling of complex dynamic systems consisting of an aggregating tractor and a front-mounted machine, the development and solution of the final form of differential equations of oscillatory motion and computer modelling of the oscillatory process were used. According to the developed equivalent scheme, a new differential equation of angular vibrations of the machine was solved in the final form. In addition, numerical modelling was performed on a PC, which allowed constructing graphical dependencies of the machine's turning angle φ at different speeds V of its forward movement and values of the c stiffness coefficient and μ damping coefficient of the pneumatic tyres of the copying wheels, and at different values of the unevenness of the soil surface and the main design parameters of the machine. It was established that with an increase V from 1.5 m·s^{-1} to 2.5 m·s^{-1}, the amplitude of oscillations of the machine's turning angle φ increased from 0.88° to 1.18°. However, at V 1.5 m·s^{-1}, the duration of the transient process is 0.22 s, and at a speed of 2.5 m·s^{-1}, this figure is already 0.14 s, i.e., a decrease of 36%. The positive amplitude of oscillations φ of the machine's steering angle reaches 1.2°, and the negative amplitude does not exceed 0.3°, i.e., the oscillation range is insignificant. In the range of values considered V, preference should be assigned to its higher value. The structural and kinematic parameters of the system examined were determined using computer simulation. The presented method of mathematical modelling of the oscillatory process can be used in the research of any machines that are hitched in front of the aggregating tractors

Keywords: sugar beet; harvesting; oscillations; differential equation; solving on a PC; graphs

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INTRODUCTION

The tops of most root crops (sugar, fodder and table roots, carrots, turnips, rutabagas, etc.), if produced in sufficient quantities, can be of significant benefit both as organic fertiliser and as livestock feed and raw material for biogas production. Therefore, cutting, collecting and using root crop tops is an urgent task in the field of agricultural mechanisation.

Since many root crops are grown using modern technologies that involve harvesting the tops of these crops on the root, the machines that will perform these operations must comply with the relevant requirements to prevent damage and loss of root crop bodies and their damage, dislodging the root crops themselves from the soil, complete harvesting of the tops and preventing their contamination with soil impurities. In addition, accurate and high-quality pruning of tops from the heads of root crops reduces the loss of their upper parts, which will significantly increase the full yield of root crops per hectare of their crops [1].

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Thus, the most common multistage method of harvesting root crop tops involves first making a continuous main cut of the entire crop, followed by harvesting and unloading into a vehicle or scattering it across the field. Next, the heads of the root crops are cleaned or trimmed (or simultaneously cleaned and trimmed with different working tools) from the remaining tops on the root.

Therefore, this sequence must be followed before removing the root crops from the soil, and the tops harvesting machines, either as independent agricultural machines or as top harvesting modules of various combines, must be attached to the front of the power vehicle (i.e., to the aggregating tractor or the front of the frame of self-propelled combines) [2].

Analysing the studies published on this subject [2-4], it can be stated that the results of experimental studies indicate that during operation, the machines for cutting tops frontally attached to the aggregating tractor perform oscillatory movements, which are determined by the relief of uneven field surfaces, the forward speed of the aggregating tractor, the location of the copy wheels relative to the machine suspension system, etc. Using the latter causes oscillatory movements of machines for harvesting root crops, which significantly worsens the quality indicators of the process in question. Namely, it is necessary to improve the quality of cutting the tops from each head of root crops over the entire working width of the machine and to collect and unload them without significant losses. And this should be the case in any condition of the location of the heads of root crops in terms of their height. It requires precise and sensitive reproduction of soil surface irregularities and the height of root crop heads.

Despite the widespread use of front-mounted machines for harvesting root crop tops, there is virtually no fundamental analytical research into their oscillatory movements over uneven soil surfaces. Works [1; 4; 5] present theoretical studies of oscillations of a machine for cutting tops of root crops frontally attached to an aggregating tractor using the equations of dynamics of mechanical systems in generalised coordinates. Therewith, these studies do not always consider some important parameters. For example, the forces acting on topsy-turvy machines are not always considered, oscillatory movements are considered in the presence of unnecessary generalised coordinates, and unjustified simplifications are made when solving the resulting differential equations. In addition, when solving this problem, a dynamic system consisting of two differential equations of oscillatory motion is explored, i.e., in the presence of an extra generalised coordinate. Having just one coordinate will be enough.

In studies [6-8], it was demonstrated that the issues of high-quality sugar beet tops harvesting are solved based on other conditions not related specifically to the oscillatory processes of top harvesting machines and their working bodies.

The issues of exploring the energy performance of harvesting root crops and processing raw materials are discussed in [9-11]. However, they do not consider the dynamics of harvesting machines. The study of the stability of a ploughing unit mounted frontally on an aggregating tractor is devoted to [12-14], the methodology for constructing graphical dependencies of oscillatory processes is given in [15-17], and the controllability of movement is reflected in [18]. The issues of changes in soil properties that affect the oscillations of harvesting machines are discussed in [19; 20]. The studies presented in well-known works provide grounds for the development of a mathematical model that would accurately describe the oscillatory process of a machine for harvesting root crops mounted in front of an aggregating tractor.

Based on the above and using the methodology discussed in [3], it is possible to build a more accurate mathematical model of the movement of a machine for cutting root crops in the longitudinal-vertical plane when it moves along uneven soil surfaces. It will allow for a more accurate study and determination of such kinematic and structural parameters, using which will contribute to a significant improvement of the technological process under consideration.

The purpose of the study was to determine the optimal structural and kinematic parameters of a machine for cutting tops of various root crops, which is attached to the front of a wheeled aggregating tractor, by analytically exploring the dynamics of its oscillatory movements in the longitudinal-vertical plane.

MATERIALS AND METHODS

The mathematical modelling was performed using the methods of constructing computational mathematical models of agricultural machines and machine units. Theoretical studies of the obtained differential equation of oscillatory motion are conducted using the basic provisions of the dynamics of complex dynamic systems based on the methods of analytical mechanics, vibration theory and higher mathematics (in particular, the method of uncertain coefficients is used to solve the differential equation). The numerical solution was performed using a personal computer and PTC Mathcad 15 software.

At the Department of Mechanics of the National University of Life and Environmental Sciences of Ukraine, and the Institute of Mechanics and Automation of Agricultural Production of the National Academy of Agrarian Sciences of Ukraine, under the guidance of Prof. V.M. Bulgakov, a new design of a machine for cutting root crop tops was developed, which works like a mower and can be used to cut tops of both many root crops on the root and perennial grasses, as it is frontally attached to the aggregating tractor [1]. Several prototypes of machines for harvesting root crop tops were manufactured in industrial conditions and tested in the field, with positive results. Comprehensive theoretical and experimental studies are conducted to optimise its structural and kinematic parameters.

This machine uses a rotary cutting unit that is positioned over the entire working width of the machine. The rotary cutting machine has arc-shaped cutting blades that are pivotally mounted on the drive drum in an appropriate pitch. The drive drum rotates in the opposite rotation to the forward motion of the entire machine. The sharp blades of the arcuate cutting knives cover the entire working width of the cutter bar. The arc-shaped cutting knives rotating in the longitudinal-vertical plane provide a free-flowing cut of the main mass of the tops along the entire working width of the machine and ensure that it is transferred to the screw conveyor, which collects and transports it to the end part of the machine. Here, using an unloading mechanism and a spigot of the appropriate height, the cut tops are unloaded into a vehicle moving alongside the harvested area of the field [1; 4; 6]. If necessary, the discharge mechanism and the nozzle can be detached from the machine, and the cut stubble is then discharged onto the harvested area of the field and used as fertiliser. The rotary cutter bar (i.e. the actual sharp ends of its arc-shaped cutting blades) is positioned at the appropriate cutting height relative to the surface established by the root crops (in the crop rows) using two tracking wheels (with screw mechanisms for their installation relative to the machine frame). The tracking wheels are positioned in the row spacing of the root crops and roll over uneven soil surfaces in the row spacing but track the general unevenness in the machine's working area. Considering the fact that root crops are harvested at the root, and the heads of these crops can be located at different heights relative to the soil surface itself, the rotary cutter is set to a higher cutting height using the screw mechanisms of the copy wheels. It prevents the bodies of the root crops themselves from being removed along with the tops, damaging them, which is extremely undesirable and knocking the root crops out of the soil. Therefore, the operations of complete and final harvesting of root crop tops are performed by other machines, in particular, root crop head cleaners for removing tops from the crops or passive (sometimes active) head trimmers.

Figure 1 presents a general overview of this machine for cutting the tops of root crops at the root.

Figure 1. Root crop peeling machine with rotary cutting unit

The qualitative performance of the machine for cutting the tops of root crops, namely the completeness of the cut, the absence of damage and the dislodging of root crops from the soil, will be determined by the stability of the movement of its rotary cutting unit in the longitudinal-vertical plane. And this will be positively influenced by the sensitivity of the machine's tracking system, which will be able to track all the unevenness of the surface of the field with root crops, and by the design parameters of the machine and the device for connecting it to the wheeled tractor. Mathematical modelling of the oscillations of a machine for cutting the tops of root crops can be performed if differential equations of its oscillatory movements in the corresponding plane are compiled. For this purpose, first of all, an equivalent diagram of this machine for cutting the tops of root crops in the longitudinal-vertical plane was built, which is presented in Figure 2.

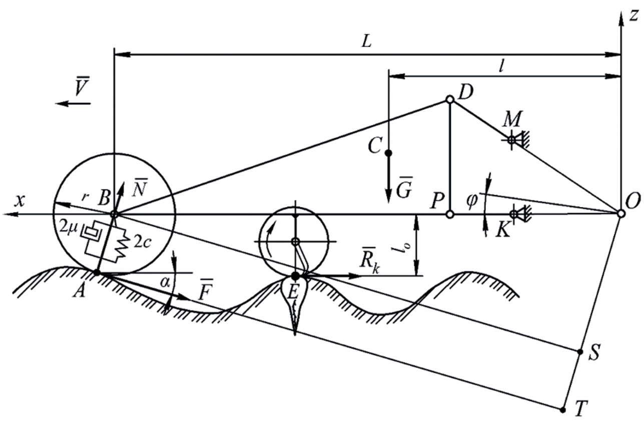

Figure 2. An equivalent diagram of a machine for cutting off the tops of root crops φ – generalised coordinate (the angle of rotation by which the machine deviates in the longitudinal-vertical plane); α – the angle of deviation of the friction force ($\bar{F}$) from the horizontal, which changes its value in different areas of the uneven ground surface on which the copy wheel moves; c – coefficient of stiffness of the pneumatic tyre of the copy wheel; μ – coefficient of damping of the pneumatic tyre of the copy wheel

The features of this equivalent arrangement are that the root crop top dresser is connected to the aggregating tractor at the front. For this purpose, two lower (*PK*) and one upper (*MD*) linkages are used, i.e. the central linkage of the tractor's front linkage. When performing the technological process of harvesting tops with a root crop tops harvester, the connecting rods of the front linkage of the implement are unlocked. It is in this position that the root crop top dresser uses its pneumatic tracking wheels, which are positioned and move in the row spacing of the root crops, to effectively follow the unevenness of the soil surface and set its rotary cutter bar to the desired cutting height. The equivalent diagram represents the movement of a root crop topping machine in the longitudinal-vertical plane when its front tracking wheel moves over an uneven soil surface that has the form of a harmonic function. Figure 2 demonstrates all the external forces acting on the pneumatic copy wheels (two wheels combined into one and represented as elastic-damping models) and the rotary cutter and provides the main linear and angular dimensions.

RESULTS AND DISCUSSION

In the mathematical modelling of the movement of the machine for cutting the tops of root crops, equations in the Lagrange form of the 2nd kind were used, all the necessary transformations were performed, and the differential equation of its movement was obtained in the following form [1; 3]:

$$A_2\cdot\ddot{\varphi}+A_1\cdot\dot{\varphi}+A_o\cdot\varphi=B_1\cdot\dot{h}+B_o\cdot h+C, \tag{1}$$

where the coefficients A_o, A_1, A_2, B_o, B_1 are constant values, which are equal to each other:

$A_2=I_{oy}$ – is the moment of inertia of the machine relative to the axis O_y (this axis is not presented in the diagram of Fig. 2), which is perpendicular to the longitudinal-vertical plane *zOx* and passes through the point *O*, kg·m²; $A_1=2\mu\cdot L^2$, (μ is the damping coefficient of the pneumatic tyre of the copying wheel); $A_o=2c\cdot L^2$, (*c* is the stiffness coefficient of the pneumatic tyre of the copying wheel); $B_1=2\mu\cdot L$; $B_o=2c\cdot L$. φ is the angle of rotation of the machine in the longitudinal-vertical plane relative to the centre of rotation (point *O*); *h* – is the ordinate of the unevenness of the soil surface.

Equation (1) belongs to the class of second-order differential equations with constant coefficients and a right-hand side.

In this case, the term *C* included in equation (1) is determined from the following expression:

$$C=N\cdot[L-f(r+L\cdot\alpha)]-R_k\cdot l_o-G\cdot l, \tag{2}$$

where *N* – unknown normal reaction of the bunch (i.e. normal reaction of the soil to the tracking wheel at point *A*); *L* – distance from the conditional point *O* of machine turning to the axis of the tracking wheels (point B); *r* – radius of the tracking wheel; α – angle of deviation of the friction force $\bar{F}$ from the horizontal, provided that $\sin\alpha=\alpha$ i.e. when the angle is small α; R_k – the total force of cutting the tops of root crops, reduced to point *E*, i.e. the end of the arcuate knife of the rotary top cutter; l_o – the height of the cut of the tops of root crops (or the distance from the end of the arcuate knife of the rotary top cutter to the lower base of the machine); *G* – the weight of the machine for cutting the tops of root crops; *l* – the distance from the conditional point *O* of rotation of the machine to its centre of mass.

Thus, equation (1) contains two unknowns, namely φ and *N*.

However, N can be determined to a first approximation from the static equilibrium equation of the mechanical system under consideration with sufficient accuracy for practice. Namely:

$$N=\frac{R_k\cdot l_o+G\cdot l}{L-f(r+L\cdot\alpha)}. \tag{3}$$

It is easy to confirm that after substituting expression (3) into expression (2), the value of *C* will turn to zero, i.e. *C*=0.

Thus, equation (1), with sufficient accuracy, is transformed into the following equation:

$$A_2\cdot\ddot{\varphi}+A_1\cdot\dot{\varphi}+A_o\cdot\varphi=B_1\cdot\dot{h}+B_o\cdot h. \tag{4}$$

Since the harmonic function *h*, which is included in the right-hand side of equation (4), determines the unevenness of the soil surface over which the machine's copy wheel moves, it is determined according to the following expression [6]:

$$h=h_o\left(1-\cos\frac{2\pi\cdot x}{l_1}\right)=h_o\left(1-\cos\frac{2\pi\cdot V\cdot t}{l_1}\right), \tag{5}$$

along with its first derivative, whose value at time t will be equal to:

$$\dot{h}=\frac{2\pi\cdot h_o\cdot V}{l_1}\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}, \tag{6}$$

where *h* – the actual current ordinate of the unevenness of the soil surface, m; h_o – the amplitude of the unevenness of the facet surface; l_1 – the period of oscillation of the unevenness of the soil surface, m; $x=V\cdot t$ – the current abscissa of the unevenness of the soil surface, m; *V* – the forward speed of the machine, m·s⁻¹, then after substituting (5) and (6) into equation (4), the following differential equation is obtained:

$$A_2\cdot\ddot{\varphi}+A_1\cdot\dot{\varphi}+A_o\cdot\varphi=\frac{2\pi\cdot B_1\cdot h_o\cdot V}{l_1}\cdot$$
$$\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}-B_o\cdot h_o\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+B_o\cdot h_o\ , \tag{7}$$

which can be converted to the following equation:

$$\ddot{\varphi}+\frac{A_1}{A_2}\cdot\dot{\varphi}+\frac{A_o}{A_2}\cdot\varphi=\frac{2\pi\cdot B_1\cdot h_o\cdot V}{A_2\cdot l_1}\cdot$$
$$\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}-\frac{B_o\cdot h_o}{A_2}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+\frac{B_o\cdot h_o}{A_2}. \tag{8}$$

Firstly, consider the general solution of the homogeneous differential equation of the following form:

$$\ddot{\varphi}+\frac{A_1}{A_2}\cdot\dot{\varphi}+\frac{A_o}{A_2}\cdot\varphi=0. \tag{9}$$

Its characteristic equation is as follows:

$$k^2+\frac{A_1}{A_2}\cdot k+\frac{A_o}{A_2}=0, \tag{10}$$

where k – roots of the characteristic equation.

Determine the roots of the quadratic equation (10). Have:

$$k_1=-\frac{A_1}{2A_2}+\sqrt{\frac{A_1^2}{4\cdot A_2^2}-\frac{A_o}{A_2}}.\quad k_2=-\frac{A_1}{2A_2}-\sqrt{\frac{A_1^2}{4\cdot A_2^2}-\frac{A_o}{A_2}}. \tag{11}$$

Natural oscillations of the mechanical system examined are possible when the roots k1 and k2 are complex numbers.

It will be the case if:

$$\frac{A_1^2}{4\cdot A_2^2}-\frac{A_o}{A_2}<0. \tag{12}$$

It can be demonstrated that within the real parameters of the machine for cutting the tops of root crops under consideration, inequality (12) is always performed. Indeed, inequality (12):

$$\frac{A_1^2}{4\cdot A_2^2}-\frac{A_o}{A_2}=\frac{A_1^2-4A_2\cdot A_o}{4\cdot A_2^2}<0, \tag{13}$$

is summarised in the following inequality:

$$A_1^2\text{-}4A_2{\cdot}A_o<0, \tag{14}$$

or, considering the expressions for the coefficients included in this inequality, obtain:

$$(2\mu{\cdot}L^2)^2\text{-}4I_{oy}{\cdot}2c{\cdot}L^2<0, \tag{15}$$

At 2μ=150 N·s·m^{-1}, I_{oy}=30 kg·m^2, $2c$=4000 N·m^{-1}, L=1.8 m, there are:

(150·1.8^2)^2-4·30·4000·1.8^2=236196-1555200=-1319004<0.

Thus, the roots of the characteristic equation (10) are always complex and equal:

$$k_1=-\frac{A_1}{2A_2}+\sqrt{\frac{A_o}{A_2}-\frac{A_1^2}{4\cdot A_2^2}}\cdot i,\quad k_2=-\frac{A_1}{2A_2}-\sqrt{\frac{A_o}{A_2}-\frac{A_1^2}{4\cdot A_2^2}}\cdot i, \tag{16}$$

where i – imaginary unit.

In this case, the general solution $\bar{\varphi}$ of the homogeneous equation (9) will be as follows:

$$\bar{\varphi}=e^{-\frac{A_1}{2A_2}t}\cdot\left[\begin{array}{l}C_1\cdot\sin\left(\sqrt{\frac{A_o}{A_2}-\frac{A_1^2}{4\cdot A_2^2}}\cdot t\right)+\\+C_2\cdot\cos\left(\sqrt{\frac{A_o}{A_2}-\frac{A_1^2}{4\cdot A_2^2}}\cdot t\right)\end{array}\right], \tag{17}$$

where C_1 and C_2 – arbitrary constants that are obtained from the initial conditions.

Next, determine the partial solution φ^* of the heterogeneous equation (8). Considering the right-hand side of the differential equation (8), its partial solution φ^* is found in the following form:

$$\varphi^*=M\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}+N\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+R, \tag{18}$$

where M, N and R – uncertain coefficients.

To find the coefficients M, N, and R, use the method of uncertain coefficients. To do this, determine the first and second derivatives of time t for expression (18).

$$\dot{\varphi}^*=M\cdot\frac{2\pi\cdot V}{l_1}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}-N\cdot\frac{2\pi\cdot V}{l_1}\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}, \tag{19}$$

$$\ddot{\varphi}^*=-M\cdot\frac{4\pi^2\cdot V^2}{l_1^2}\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}-N\cdot\frac{4\pi^2\cdot V^2}{l_1^2}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}. \tag{20}$$

Substituting expressions (18) and (19) into the differential equation (8), obtain:

$$\begin{gathered}-M\cdot\frac{4\pi^2\cdot V^2}{l_1^2}\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}-N\cdot\\ \cdot\frac{4\pi^2\cdot V^2}{l_1^2}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+\frac{A_1}{A_2}\cdot\\ \cdot M\cdot\frac{2\pi\cdot V}{l_1}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}-\frac{A_1}{A_2}\cdot N\cdot\frac{2\pi\cdot V}{l_1}\cdot\\ \cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}+\frac{A_o}{A_2}\cdot M\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}+\frac{A_o}{A_2}\cdot N\cdot\\ \cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+\frac{A_o}{A_2}\cdot R=\frac{2\pi\cdot B_1\cdot h_o\cdot V}{A_2\cdot l_1}\cdot\\ \cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}-\frac{B_o\cdot h_o}{A_2}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+\frac{B_o\cdot h_o}{A_2}.\end{gathered} \tag{21}$$

By equating the coefficients of the same trigonometric functions and the free terms in the left and right sides of expression (21), obtain the following system of linear algebraic equations with respect to the unknowns M, N, and R:

$$\left.\begin{array}{l}\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)\cdot M-\frac{2\cdot A_1\cdot\pi\cdot V}{A_2\cdot l_1}\cdot N=\frac{2\pi\cdot B_1\cdot h_o\cdot V}{A_2\cdot l_1},\\ \frac{2\cdot A_1\cdot\pi\cdot V}{A_2\cdot l_1}\cdot M+\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)\cdot N=\frac{B_o\cdot h_o}{A_2},\\ \frac{A_o}{A_2}\cdot R=\frac{B_o\cdot h_o}{A_2}.\end{array}\right\} \tag{22}$$

From the third equation of system (22), the unknown R is determined:

$$R=\frac{B_o\cdot h_o}{A_o}. \tag{23}$$

To reduce the cumbersomeness of the equations, the following notation will be used:

$$-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}=S, \tag{24}$$

$$\frac{2\cdot A_1\cdot\pi\cdot V}{A_2\cdot l_1}=T, \tag{25}$$

$$\frac{2\cdot\pi\cdot B_1\cdot h_o\cdot V}{A_2\cdot l_1}=Q, \tag{26}$$

$$-\frac{B_o\cdot h_o}{A_2}=K. \tag{27}$$

Then the system of the first two equations of system (22) will be written in the following form:

$$\left.\begin{array}{l}S\cdot M-T\cdot N=Q,\\ T\cdot M+S\cdot N=K.\end{array}\right\} \tag{28}$$

Determine the principal determinant of the system (28):

$$\Delta=\begin{vmatrix}S & -T\\ T & S\end{vmatrix}=S^2+T^2\neq 0. \tag{29}$$

Therefore, to find the unknowns M and N from the system of equations (28), the Kramer rule can be applied. Calculate the necessary determinants:

$$\Delta_M = \begin{vmatrix} Q & -T \\ K & S \end{vmatrix} = Q \cdot S + K \cdot T, \quad (30)$$

and

$$\Delta_N = \begin{vmatrix} S & Q \\ T & K \end{vmatrix} = S \cdot K - T \cdot Q. \quad (31)$$

Then:

$$M = \frac{\Delta_M}{\Delta} = \frac{Q \cdot S + K \cdot T}{S^2 + T^2}, \quad (32)$$

and

$$N = \frac{\Delta_N}{\Delta} = \frac{S \cdot K - T \cdot Q}{S^2 + T^2}. \quad (33)$$

Substituting expressions (24)-(27) into expressions (32) and (33), accordingly, obtain:

$$M = \frac{\frac{2\pi \cdot B_1 \cdot h_o \cdot V}{A_2 \cdot l_1} \cdot \left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right) - \frac{2B_o \cdot h_o \cdot A_1 \cdot \pi \cdot V}{A_2^2 \cdot l_1}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}}, \quad (34)$$

and

$$N = \frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}}. \quad (35)$$

Considering (34) and (35), the partial solution (18) of the differential equation (8) will be as follows:

$$\varphi^* = \frac{\frac{2\pi \cdot B_1 \cdot h_o \cdot V}{A_2 \cdot l_1} \cdot \left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right) - \frac{2B_o \cdot h_o \cdot A_1 \cdot \pi \cdot V}{A_2^2 \cdot l_1}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot$$
$$\cdot \sin \frac{2\pi \cdot V \cdot t}{l_1} + \frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot$$
$$\cdot \cos \frac{2\pi \cdot V \cdot t}{l_1} + \frac{B_o \cdot h_o}{A_o}. \quad (36)$$

Then the total solution of the differential equation with the right-hand side of (8), considering (17) and (36), will be equal:

$$\varphi = \bar{\varphi} + \varphi^* = e^{-\frac{A_1}{2A_2} \cdot t} \cdot \left[C_1 \cdot \sin\left(\sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot t\right) + C_2 \cdot \cos\left(\sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot t\right) \right] +$$
$$+ \frac{\frac{2\pi \cdot B_1 \cdot h_o \cdot V}{A_2 \cdot l_1} \cdot \left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right) - \frac{2B_o \cdot h_o \cdot A_1 \cdot \pi \cdot V}{A_2^2 \cdot l_1}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot \sin \frac{2\pi \cdot V \cdot t}{l_1} +$$
$$+ \frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot \cos \frac{2\pi \cdot V \cdot t}{l_1} + \frac{B_o \cdot h_o}{A_o}. \quad (37)$$

To determine the arbitrary constants C_1 and C_2, the following initial conditions are set: since the motion starts from a state of rest, i.e. at t=0, then:

$$\varphi=0, \dot{\varphi}=0.$$

Next, find the first derivative of expression (37) in time t. Have:

$$\varphi = \dot{\bar{\varphi}} + \dot{\varphi}^* = -\frac{A_1}{2A_2} \cdot e^{-\frac{A_1}{2A_2} \cdot t} \cdot \left[C_1 \cdot \sin\left(\sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot t\right) + \right.$$
$$\left. + C_2 \cdot \cos\left(\sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot t\right) \right] + e^{-\frac{A_1}{2A_2} \cdot t} \cdot$$
$$\cdot \left[C_1 \cdot \sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot \cos\left(\sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot t\right) - \right.$$
$$\left. - C_2 \cdot \sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot \sin\left(\sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} \cdot t\right) \right] + \quad (38)$$
$$+ \frac{\frac{2\pi \cdot B_1 \cdot h_o \cdot V}{A_2 \cdot l_1} \cdot \left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right) - \frac{2B_o \cdot h_o \cdot A_1 \cdot \pi \cdot V}{A_2^2 \cdot l_1}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot \frac{2\pi \cdot V}{l_1} \cdot$$
$$\cdot \cos \frac{2\pi \cdot V \cdot t}{l_1} - \frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot$$
$$\cdot \frac{2\pi \cdot V}{l_1} \cdot \sin \frac{2\pi \cdot V \cdot t}{l_1}.$$

Considering the initial conditions, the following expression is obtained from (37):

$$C_2 + \frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} + \frac{B_o \cdot h_o}{A_o} = 0. \quad (39)$$

Under the same initial conditions, the following is obtained from (38):

$$-\frac{A_1}{2A_2} \cdot C_1 + C_2 \cdot$$
$$\cdot \sqrt{\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}} + \frac{\frac{2\pi \cdot B_1 \cdot h_o \cdot V}{A_2 \cdot l_1} \cdot \left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right) - \frac{2B_o \cdot h_o \cdot A_1 \cdot \pi \cdot V}{A_2^2 \cdot l_1}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot \quad (40)$$
$$\cdot \frac{2\pi \cdot V}{l_1} = 0.$$

From expression (39), the value of the arbitrary constant C_2 is obtained:

$$C_2 = -\frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} - \frac{B_o \cdot h_o}{A_o}. \quad (41)$$

Substituting (41) into (40), the value of the arbitrary constant C_1 is obtained:

$$C_1 = \left\{ \frac{A_1}{2A_2} \left[-\frac{\left(\frac{4\pi^2 \cdot V^2}{l_1^2} - \frac{A_o}{A_2}\right) \cdot \frac{B_o \cdot h_o}{A_2} - \frac{4A_1 \cdot B_1 \cdot \pi^2 \cdot h_o \cdot V^2}{A_2^2 \cdot l_1^2}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} - \frac{B_o \cdot h_o}{A_o} \right] - \right.$$
$$\left. - \frac{\frac{2\pi \cdot B_1 \cdot h_o \cdot V}{A_2 \cdot l_1} \cdot \left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right) - \frac{2B_o \cdot h_o \cdot A_1 \cdot \pi \cdot V}{A_2^2 \cdot l_1}}{\left(-\frac{4\pi^2 \cdot V^2}{l_1^2} + \frac{A_o}{A_2}\right)^2 + \frac{4A_1^2 \cdot \pi^2 \cdot V^2}{A_2^2 \cdot l_1^2}} \cdot \frac{2\pi \cdot V}{l_1} \right\} \times \left(\frac{A_o}{A_2} - \frac{A_1^2}{4A_2^2}\right)^{-\frac{1}{2}}. \quad (42)$$

Substituting expressions (41) and (42) into the general solution (37) of equation (8), a partial solution of this equation satisfying the initial conditions is obtained:

$$\begin{aligned}\varphi = e^{-\frac{A_1}{2A_2}\cdot t}\cdot\Bigg\{\Bigg\{\frac{A_1}{2A_2}\Bigg[-\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{A_o}{A_2}\right)\cdot\frac{B_o\cdot h_o}{A_2}-\frac{4A_1\cdot B_1\cdot\pi^2\cdot h_o\cdot V^2}{A_2^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)^2+\frac{4A_1^2\cdot\pi^2\cdot V^2}{A_2^2\cdot l_1^2}}-\frac{B_o\cdot h_o}{A_o}\Bigg]-\frac{\frac{2\pi\cdot B_1\cdot h_o\cdot V}{A_2\cdot l_1}\cdot\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)-\frac{2B_o\cdot h_o\cdot A_1\cdot\pi\cdot V}{A_2^2\cdot l_1}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)^2+\frac{4A_1^2\cdot\pi^2\cdot V^2}{A_2^2\cdot l_1^2}}\cdot\frac{2\pi\cdot V}{l_1}\Bigg\}\cdot\\ \cdot\left(\frac{A_o}{A_2}-\frac{A_1^2}{4A_2^2}\right)^{-\frac{1}{2}}\cdot\sin\left(\sqrt{\frac{A_o}{A_2}-\frac{A_1^2}{4A_2^2}}\cdot t\right)-\Bigg[-\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{A_o}{A_2}\right)\cdot\frac{B_o\cdot h_o}{A_2}-\frac{4A_1\cdot B_1\cdot\pi^2\cdot h_o\cdot V^2}{A_2^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)^2+\frac{4A_1^2\cdot\pi^2\cdot V^2}{A_2^2\cdot l_1^2}}+\frac{B_o\cdot h_o}{A_o}\Bigg]\cdot\cos\left(\sqrt{\frac{A_o}{A_2}-\frac{A_1^2}{4A_2^2}}\cdot t\right)\Bigg\}+\\ +\frac{\frac{2\pi\cdot B_1\cdot h_o\cdot V}{A_2\cdot l_1}\cdot\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)-\frac{2B_o\cdot h_o\cdot A_1\cdot\pi\cdot V}{A_2^2\cdot l_1}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)^2+\frac{4A_1^2\cdot\pi^2\cdot V^2}{A_2^2\cdot l_1^2}}\cdot\sin\frac{2\pi\cdot V\cdot t}{l_1}+\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{A_o}{A_2}\right)\cdot\frac{B_o\cdot h_o}{A_2}-\frac{4A_1\cdot B_1\cdot\pi^2\cdot h_o\cdot V^2}{A_2^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{A_o}{A_2}\right)^2+\frac{4A_1^2\cdot\pi^2\cdot V^2}{A_2^2\cdot l_1^2}}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+\frac{B_o\cdot h_o}{A_o}.\end{aligned} \tag{43}$$

And finally, substituting the expressions for the coefficients A_o, A_1, A_2, B_o, B_1 in (43), finally obtaining a partial solution φ of the differential equation (8):

$$\begin{aligned}\varphi = e^{-\frac{\mu\cdot L^2}{I_{oy}}\cdot t}\Bigg\{\Bigg\{\frac{\mu\cdot L^2}{I_{oy}}\cdot\Bigg[-\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{2c\cdot L^2}{I_{oy}}\right)\cdot\frac{2c\cdot L\cdot h_o}{I_{oy}}-\frac{16\mu^2\cdot L^3\cdot\pi^2\cdot h_o\cdot V^2}{I_{oy}^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}-\frac{h_o}{L}\Bigg]-\frac{\frac{4\pi\cdot\mu\cdot L\cdot h_o\cdot V}{I_{oy}\cdot l_1}\cdot\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)-\frac{8c\cdot L^3\cdot h_o\cdot\mu\cdot\pi\cdot V}{I_{oy}^2\cdot l_1}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}\cdot\frac{2\pi\cdot V}{l_1}\Bigg\}\left(\frac{2c\cdot L^2}{I_{oy}}-\frac{\mu^2\cdot L^4}{I_{oy}^2}\right)^{-\frac{1}{2}}\cdot\\ \sin\left(\sqrt{\frac{2c\cdot L^2}{I_{oy}}-\frac{\mu^2\cdot L^4}{I_{oy}^2}}\cdot t\right)-\Bigg[\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{2c\cdot L^2}{I_{oy}}\right)\cdot\frac{2c\cdot L\cdot h_o}{I_{oy}}-\frac{16\mu^2\cdot L^3\cdot\pi^2\cdot h_o\cdot V^2}{I_{oy}^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}+\frac{h_o}{L}\Bigg]\cdot\cos\left(\sqrt{\frac{2c\cdot L^2}{I_{oy}}-\frac{\mu^2\cdot L^4}{I_{oy}^2}}\cdot t\right)\Bigg\}+\frac{\frac{4\pi\cdot\mu\cdot L\cdot h_o\cdot V}{I_{oy}\cdot l_1}\cdot\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)-\frac{8c\cdot L^3\cdot h_o\cdot\mu\cdot\pi\cdot V}{I_{oy}^2\cdot l_1}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}\cdot\\ \sin\frac{2\pi\cdot V\cdot t}{l_1}+\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{2c\cdot L^2}{I_{oy}}\right)\cdot\frac{2c\cdot L\cdot h_o}{I_{oy}}-\frac{16\mu^2\cdot L^3\cdot\pi^2\cdot h_o\cdot V^2}{I_{oy}^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}\cdot\cos\frac{2\pi\cdot V\cdot t}{l_1}+\frac{h_o}{L}.\end{aligned} \tag{44}$$

The amplitude of the free (natural) oscillations of the dynamical system examined is determined from the expression:

$$A = e^{-\frac{\mu\cdot L^2}{I_{oy}}\cdot t}\cdot\sqrt{C_1^2+C_2^2},\ \text{Rad}, \tag{45}$$

where C_1 and C_2 can be determined from expressions (41) and (42), respectively. In the final expression (44), C_1 and C_2 – the coefficients of functions $\sin\left(\sqrt{\frac{2c\cdot L^2}{I_{oy}}-\frac{\mu^2\cdot L^4}{I_{oy}^2}}\cdot t\right)$ and $\cos\left(\sqrt{\frac{2c\cdot L^2}{I_{oy}}-\frac{\mu^2\cdot L^4}{I_{oy}^2}}\cdot t\right)$.

Therewith:

$$k = \sqrt{\frac{2c\cdot L^2}{I_{oy}}-\frac{\mu^2\cdot L^4}{I_{oy}^2}}, \tag{46}$$

is the circular frequency of natural oscillations of the dynamic system considered, s^{-1}.

As can be seen from expression (45), these fluctuations are damped with time t, since the factor before the square root in this expression tends to zero, i.e: $\lim_{t\to\infty} e^{-\frac{\mu\cdot L^2}{I_{oy}}\cdot t} = 0$.

The amplitude B of the forced oscillations arising from the influence of soil surface irregularities on the movement of the root crop tops cutter is equal to the root of the square of the sum of the squared coefficients in expression (44) for functions $\sin\frac{2\pi\cdot Vt}{l_1}$ and $\cos\frac{2\pi\cdot Vt}{l_1}$, respectively. This amplitude B of forced oscillations is measured in radians. Based on the above, the analytical expression for determining B is as follows:

$$B = \sqrt{\left[\frac{\frac{4\pi\cdot\mu\cdot L\cdot h_o\cdot V}{I_{oy}\cdot l_1}\cdot\left(-\frac{4\pi^2\cdot V^2}{l_1^2}+\frac{2c\cdot L^2}{I_{oy}}\right)-\frac{8c\cdot L^3\cdot h_o\cdot\mu\cdot\pi\cdot V}{I_{oy}^2\cdot l_1}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}\right]^2+\left[\frac{\left(\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{2c\cdot L^2}{I_{oy}}\right)\cdot\frac{2c\cdot L\cdot h_o}{I_{oy}}-\frac{16\mu^2\cdot L^3\cdot\pi^2\cdot h_o\cdot V^2}{I_{oy}^2\cdot l_1^2}}{\left(-\frac{4\pi^2\cdot V^2}{l_1^2}-\frac{2c\cdot L^2}{I_{oy}}\right)^2+\frac{16\mu^2\cdot L^4\cdot\pi^2\cdot V^2}{I_{oy}^2\cdot l_1^2}}\right]^2}. \tag{47}$$

The circular frequency of forced oscillations of the dynamic system considered will be equal to:

$$\nu = \frac{2\pi\cdot V}{l_1},\ s^{-1}. \tag{48}$$

Thus, finally, based on the analytical study, a closed-form solution of the differential equation of motion of the machine for cutting the tops of root crops in the longitudinal-vertical plane was performed and specific expressions were obtained to determine the main indicators of the oscillatory process. Namely, the rotation angle φ, the amplitudes of natural A and forced B oscillations and the natural frequency k and ν – the forced frequency.

Using the final analytical expressions obtained with a personal computer and the PTC Mathcad 15 software developed for this purpose, numerical calculations were performed, and new graphical dependencies were built based on them, as presented in Figure 3-Figure 7.

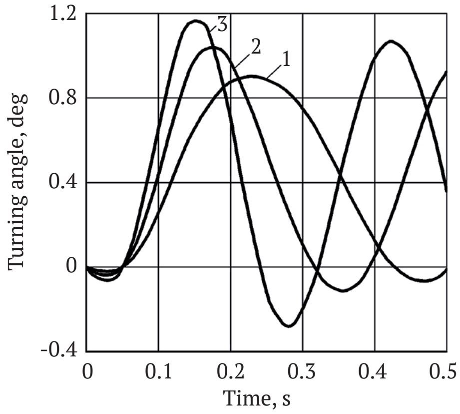

Figure 3. Dynamics of fluctuations in the angle φ of rotation of a machine for cutting tops of root crops at different speeds V of its working movement: 1 – 1.5 m·s^{-1}; 2 – 2.0 m·s^{-1}; 3 – 2.5 m·s^{-1}

Thus, the analysis of the mathematical modelling data, represented by the curves in Figure 3, demonstrates that a change in the speed V of the working translational movement of the machine for cutting the tops of root crops has two consequences.

Thus, the essence of the first consequence is that when the speed V of the translational movement of the machine increases from 1.5 to 2.5 m·s^{-1}, the amplitude of oscillations of the angle φ of rotation of the machine for cutting the tops of root crops in the examined plane undesirably increases (Fig. 3). The first maximum of this process indicates that this increase is 0.3°: from 0.88° at a travel speed of V=1.5 m·s^{-1} (curve 1 on the graph) to 1.18o, when the forward speed V of the machine for cutting the tops of root crops at the root is already 2.5 m·s^{-1} (curve 3 on the graph).

The second consequence is related to the dynamics of changes in such an estimation parameter as the time when the considered dynamic system reaches the first maximum of the oscillations of angle φ. In this case, this time can be assumed to be the duration of the transitional process of the system's reaching a stable mode of operation. The analysis of the curves in Figure 3 demonstrates that when the machine for cutting root crop tops moves with a translational speed of V, equal to 1.5 m·s^{-1}, the duration of the transient process is 0.22 s (curve 1 on the graph). During the translational movement of the machine for cutting the tops of root crops with a movement speed of V, equal to 2.5 m·s^{-1}, the value of the estimated indicator decreases to 0.14 s, i.e. by 36%, which is desirable.

As can be observed from the graphs of Figure 3, the positive amplitude of oscillations of the angle φ of rotation of the machine for cutting the tops of root crops in the investigated plane is no more than 1.2°, and the negative amplitude does not even reach 0.3°. As a result, the oscillation range of the considered oscillatory process is insignificant and does not exceed 1.4°.

It indicates that in the considered range of speeds of the V machine for cutting the tops of root crops, equal to 1.5...2.5 m·s^{-1}, preference should be given to higher values of this parameter. Therewith, consider that an increase in the speed of movement V of this machine will ensure the appropriate performance, albeit with a slight increase in the amplitude of angular oscillations and a small (in absolute value) reduction in the transient process, which will bring this dynamic system to a stable mode of operation, which will be extremely desirable.

Therewith, the negative amplitudes of the oscillations of the angle φ (refer to the graphs in Fig. 3) are conditioned upon the elastic properties of the copy wheels, which are quantified by the value of the coefficient c of the stiffness of their pneumatic tyres. Therewith, it is observed that the smaller it is, the more noticeable the response of the root crop cutting machine (through its support and copying wheels) in response to fluctuations in the longitudinal profile of field surface irregularities represented by dependence (5).

Further analysis of the mathematical modelling results fully confirms this. Setting the stiffness coefficient of the c pneumatic tyres of the copy wheels of the root crop tops cutter at 550 kN·m causes its angular oscillatory movements with smaller negative and larger positive amplitudes (curve 2 in Fig. 4).

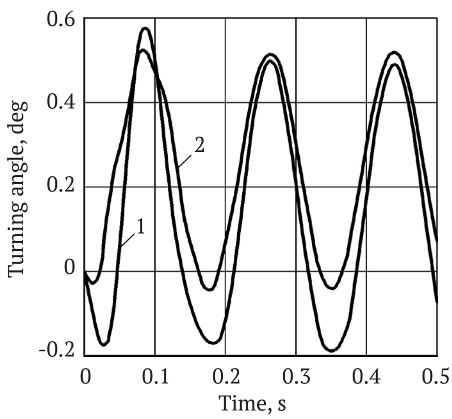

Figure 4. Dynamics of fluctuations in the angle φ of rotation of a machine for cutting tops of root crops at different values of the stiffness coefficient c of its copy wheels: 1 – 115 kN·m; 2 – 550 kN·m

In contrast, when the stiffness c of the pneumatic tyres of the machine's copy wheels is 115 kN·m, the part of the amplitude curve in the positive zone of this graph (curve 1 of Fig. 4) decreases. The negative component of the amplitude increases more significantly: by a factor of 7-8 compared to the situation with a tyre stiffnessc of 550 kN·m.

The damping coefficient μ of the tyres of the root crop top dresser is as follows. Thus, increasing the value of this parameter from 350 to 1350 kN·s·m^{-1} has very little effect on the change in the negative component of the angle oscillation amplitude φ, which for both values of the coefficient μ does not exceed 0.2° (see Fig. 5).

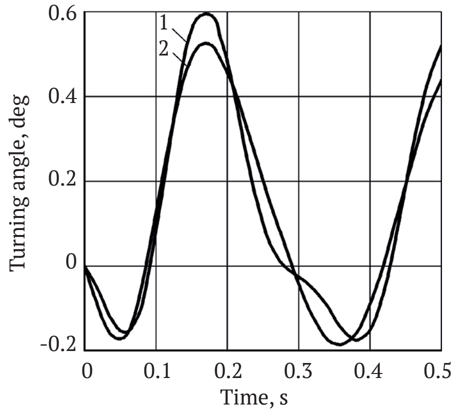

Figure 5. Dynamics of oscillations of the angle φ of rotation of a machine for cutting tops of root crops at different values of the damping coefficient μ of its copy wheels:1 – 350 kN·s·m^{-1}; 2 – 1350 kN·s·m^{-1}

As for the positive component of the amplitude of the process examined, it is 15...17% lower in the case of a larger parameter value μ (curve 2 in Fig. 5). Thus, the stiffer the tyres of the root crop peelers, the smaller the angle φ (in the positive zone of this graph) of deviation from the static position. Therewith, considering the relatively small value of this deviation, it can be stated that the machine for cutting tops of root crops is invariant to changes in the values of the damping coefficient μ of the tyres of its support and copying wheels.

Thus, there is every reason to consider this machine for cutting the tops of root crops as a dynamic system that can be represented by a physical pendulum that makes angular oscillations relative to the point O (Fig. 2). These angular oscillations are specifically determined by the amplitude of the vertical movement of the machine's copy wheels (point B in the diagram in Fig. 2). However, point B, which performs these oscillations, has an amplitude that depends entirely on the length L. Thus, it is the length that connects point B with the centre of instantaneous rotation (point O) of the oscillatory system considered.

Therewith, a change in the design parameter L causes a simultaneous change in the parameter l, which is the longitudinal coordinate of the centre of mass of the root crop top cutter (point C in Fig. 2). Thus, it is necessary to consider a mutually agreed change in the parameters L and l.

Finally, based on the calculations, it was found that an increase in the distance L (and, therewith, l; denoted as $L(l)$ in the graphs) leads to a desirable and logical decrease in the amplitude of angular oscillations of the root crop tops machine (Fig. 6). Moreover, these changes mainly relate to amplitude values that are in the positive range. From the curves presented in these graphs, it can be seen that its first maxima indicate that an increase in the values of the parameters L and l by 1 m causes a decrease in the angle φ from 0.72 to 0.44°, i.e. a decrease of almost 39%.

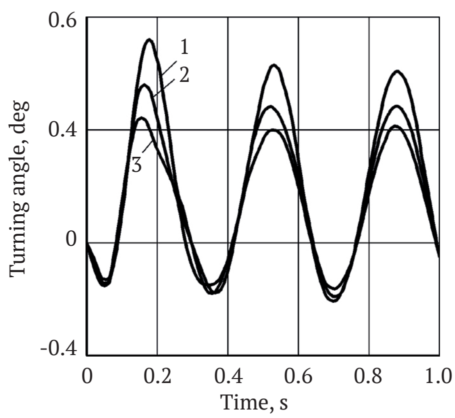

Figure 6. Dynamics of oscillations of the angle φ of rotation of the machine for cutting the tops of root crops at different values of parameters $L(l)$: 1 – 3.8(2.8) m; 2 – 4.3(3.3) m; 3 – 4.8(3.8) m

This result indicates the necessity of designing a machine for cutting the tops of root crops with the highest possible value of the *L* parameter. It can be accomplished by connecting the machine to the tractor in such a way that $\angle MDP$ (see Fig. 2) of the central linkage *MD* of the front hitch is as high as possible concerning the vertical. It should be noted that the value of the *DP* parameter must in any case be greater than the value of the *MK* parameter, which is the shortest distance between the attachment points of the centre and lower links of the front hitch to the implement. With *DP*=*MK*, the latter becomes a parallelogram, thus lifting the mounted front machine into the transport position without turning it in the longitudinal-vertical plane. From the standpoint of aggregation, it is an undesirable outcome. The variant when *DP*<*MK* is completely unacceptable, since in this case it becomes impossible to raise the frontal stubble harvester (and other trailed agricultural machines) to the transport position at all [7; 19].

In addition, the factors that affect the dynamics of the angle φ of rotation of the machine for cutting the tops of root crops include the period of fluctuations in the longitudinal profile of unevenness of the field surface l_1. The most common values of this parameter for fields where root crops are grown are 0.70 and 1.75 m.

The analysis of the calculations demonstrates that an increase in the value of the period l_1 of disturbing oscillations from 0.70 to 1.75 m, although insignificant, reduces the amplitude of angular oscillations of the machine for cutting root crop tops (Fig. 7).

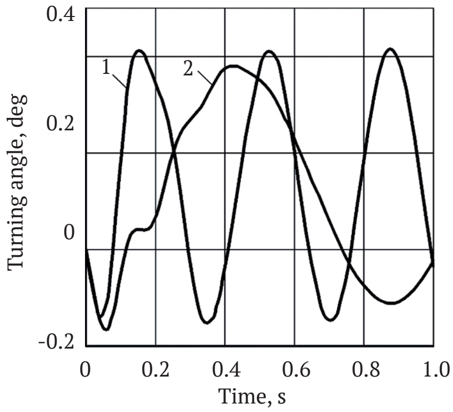

Figure 7. Dynamics of oscillations of the angle φ of rotation of the machine for cutting tops of root crops at different values of parameter l_1:1 – 0.70 m; 2 – 1.75 m

It follows that this desired reduction will be greater the larger the value of the parameter l_1. It can be achieved by carefully levelling the longitudinal profile of the field in preparation for sowing root crops.

Other calculations using a personal computer demonstrated that the influence of different values of the remaining design parameters on the dynamics of angular vibrations of the machine for cutting the tops of root crops is insignificant.

Thus, the results of the analytical study indicate the achievement of the goal, which was related to determining the optimal kinematic and design parameters of a new machine for cutting root crop tops on the root by exploring the dynamics of its oscillatory movements in the longitudinal-vertical plane.

The previously obtained results are thematically close to the present study and published scientific works of other researchers [1; 2; 4], which contain the constructed mathematical models of frontally mounted stubble harvesters on an aggregating tractor. What is common in this, and these published works is that oscillations of soil surface irregularities in the form of a harmonic function are used as an input external forced disturbance. Another common feature is the representation of the copy wheels of buncher machines in the form of elastic-damping models.

If comparing the differences between the results obtained in this research and the subject of the study and the results of other authors [1; 2; 4], it can be observed that the fundamental difference is related to the fact that the dynamics of oscillatory motion is considered here only in the presence of a single generalised coordinate φ. Thus, the absence of unnecessary generalised coordinates significantly improves the accuracy of numerical calculations on a personal computer. It is known that the study of oscillatory movements, provided that they are added by two extra coordinates related to each other, actually reflects the true nature of the processes occurring with distortions. It significantly reduces the accuracy of the results. In addition, numerical calculations on a personal computer were performed in previously published works under the assumption of constant values of many parameters, which may be of a statistical (probabilistic) nature.

None of the works [1; 2; 4], and works close to this subject [7-9], do not provide detailed transformations and the final solution of the resulting differential equation in closed form, i.e., obtaining the final solution in the form of new ordinary linear dependencies, which allow for high accuracy to conduct relatively simple numerical calculations in the future and do it for a wide class of stubble harvesting and other agricultural machines (reapers, mowers, shredders of plant residues, etc.), of various designs and rows, which are hung in front of the aggregating tractor.

The subject of research contained in works [12; 14; 15; 21] and concerning related issues presented in this research, generally relate to the quality of technological processes of mechanised harvesting of root crops, which confirms the importance of this study, since the design and kinematic parameters of the machine for harvesting root crop tops on the root obtained here only improve the process.

In addition, the methodology presented in this research for solving a differential equation that belongs to the class of second-order differential equations with constant coefficients and a right-hand side that is not available in any previously published works can be widely used to explore the dynamics of front- and rear-mounted multiple structures of various agricultural machines as part of machine-tractor units.

CONCLUSIONS

If the machine for cutting root crop tops moves at a forward speed of up to 2.5 $m \cdot s^{-1}$, the time for this oscillatory system to reach a stable mode of operation will be reduced to 0.14 s. This circumstance will contribute both to an increase in the productivity of the tops harvesting and to a slight increase in the amplitude of its angular oscillations.

The coefficient c of stiffness of the tyres of the copy wheels of a root crop peeling machine, if reduced, causes angular vibrations of the machine with slightly higher positive and significantly lower negative amplitudes in the longitudinal-vertical plane.

It was established that the stiffer the pneumatic tyres of the copying wheels of the machine for cutting the tops of root crops, the smaller the angle φ of deviation from its static position. However, the relative smallness of this deviation allows stating that the machine for cutting tops of root crops on the root is invariant to changes in the values of the coefficient μ of damping of pneumatic tyres of its copy wheels.

Reducing the amplitude of angular oscillations in the longitudinal-vertical plane of the root crop tops cutter can be achieved by increasing the distance L from the axles of the tracing wheels (point B) to point O, which is the centre of rotation of the front linkage of the aggregating tractor. Therewith, the central linkage MD of the tractor's front linkage should be inclined at the greatest possible angle to the vertical position.

A more stable movement in the longitudinal-vertical plane of the root crop top dresser can be achieved by increasing the periods l_1 of oscillation of the longitudinal profile of the uneven soil surface. And it is possible due to a more thorough levelling of the field surface, with surface tillage, where root crops will be grown.

Further research should be focused on determining the parameters of oscillatory movements of the rotary cutting unit of the machine for cutting root crop tops (possibly its springing relative to the main frame of the machine), which will increase the number of tops harvested and the quality of cutting root crop heads. In addition, it is necessary to further consider the automation of the copying process, the accuracy and reliability of obtaining information on soil surface irregularities in the rows of crops, with the possibility of further obtaining and using the information on the location of root crop heads relative to the soil surface in height.

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Математичне моделювання коливань машини для зрізання гички коренеплідних культур

Анотація. Високоякісне зрізання гички коренеплідних культур на корені обумовлює не тільки збирання її високого врожаю, а й визначає кондиційні властивості коренеплодів перед їх викопуванням з ґрунту. Тому пошук умов, які будуть це забезпечувати є важливою і актуальною проблемою виробництва цих культур. Метою даної статті було визначення шляхом аналітичного дослідження оптимальних параметрів нової конструкції машини для збирання гички коренеплідних культур, яка навішується попереду агрегатуючого трактора і здійснює коливальні рухи у повздовжньо-вертикальній площині. Були використані методи, які пов'язані з моделюванням складних динамічних систем, що складаються з агрегатуючого трактора та передньонавішеної машини, складання та розв'язування в кінцевому вигляді диференційних рівнянь коливального руху та комп'ютерного моделювання коливального процесу. Згідно розробленої еквівалентної схеми було розв'язане у кінцевому вигляді нове диференціальне рівняння кутових коливань машини. Здійснено також числове моделювання на ПК, яке дало можливість побудувати графічні залежності кута φ повороту машини за різної швидкості V її поступального руху та значень коефіцієнта c жорсткості та коефіцієнта μ демпфірування пневматичних шин копіювальних коліс, а також за різних значень нерівностей поверхні ґрунту і основних конструктивних параметрів машини. Було встановлено, що при збільшенні V з 1,5 м·с$^{-1}$ до 2,5 м·с$^{-1}$1 амплітуда коливань кута φ повороту машини зростає з 0,88° до 1,18°. Однак, при V 1,5 м·с$^{-1}$ тривалість перехідного процесу становить 0,22 с, а за швидкістю 2.5 м·с$^{-1}$ – 1 цей показник вже становить 0,14 с, тобто зменшується на 36 %. Позитивна амплітуда коливань кута φ повороту машини досягає 1,2°, а від'ємна амплітуда не перевищує 0,3°, тобто розмах коливань є незначним. У діапазоні розглянутих V перевагу слід надавати її більшим значенням. Шляхом моделювання на ПК визначені конструктивні й кінематичні параметри досліджуваної системи. Наведена методика математичного моделювання коливального процесу може бути використаною при дослідженні будь-яких машин, які начіплюються попереду агрегатуючого трактора

Ключові слова: буряк цукровий; збирання; коливання; диференціальне рівняння; розв'язування на ПК; графіки