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Serhii Pylypaka
Doctor of Technical Sciences, Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0002-1496-4615

Tetiana Volina*
PhD in Technical Sciences, Associate Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0001-8610-2208

Vyacheslav Hropost
Postgraduate Student
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0001-9363-3955

Olena Kozlova
PhD in Pedagogical Sciences, Professor
Sumy State Pedagogical University named after A.S. Makarenko
40002, 87 Romenska Str., Sumy, Ukraine
https://orcid.org/0000-0003-1626-5188

Oleksandr Tatsenko
Senior Lecturer
Sumy National Agrarian University
40021, 160 H. Kondratiev Str., Sumy, Ukraine
https://orcid.org/0000-0003-1762-8219

Investigation of deformation of the spring tooth of agricultural implements from the action of the force applied to it

Abstract. The relevance of the study lies in the need to investigate the dependence of the force applied to the spring tooth on its parameters, which is an important task due to the widespread use of spring teeth in agricultural implements, such as balers, reapers, rakes, etc. The purpose of the study is to establish an analytical description of the spring tooth deformation depending on the amount of applied force. For this purpose, the theory of bending rods from the resistance of materials was applied, without simplifying it, as is common in construction, where the deflection of a beam is small compared to its length. The calculation is based on the well-known dependence of the curvature of the elastic axis of the beam (tooth) on the applied moment and the stiffness of its cross-section. The study considers a cantilevered tooth, which at the point of pinching is a spring with several turns, followed by a smooth transition to a rectilinear shape. The

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*Corresponding author

tooth is divided into two parts along its length: curvilinear and rectilinear. Calculation of the deformation, i.e., finding the shape of the elastic axis after the action of the applied force, is carried out for both parts separately. The need for this approach is dictated by the fact that the curvature of the elastic axis of the tooth in the free state changes abruptly from the stable value of the curvilinear part to a zero value of the straight part. The main result of the study is to find the shape of the elastic axis of individual parts of the tooth under the action of the applied force and combine them into one whole. This helps to determine the amount of movement of the free end of the tooth depending on the amount of force applied to it. The application of the obtained data can help in the development of more efficient and productive agricultural tools, and increase their durability and efficiency when interacting with the soil

Keywords: curvature; elastic axis; applied force; moment

INTRODUCTION

In agriculture, in particular in the mechanisation of processes, it is important to understand and improve the parameters of agricultural tools. The effectiveness of these tools is directly related to the reliability of their components. The investigation of spring tooth deformation due to applied force determines not only the stability and efficiency of tools, but can also help improve their design and functionality. Tools with optimised spring teeth can provide increased productivity and reduce wear, which are key factors in modern farming. The use of new technological solutions in agriculture can help to increase yields, reduce resource costs, and improve the environmental sustainability of agricultural activities. Thus, the study is an important step towards optimising and modernising agricultural production.

The results of the study of the influence of operational factors on the technical condition of agricultural machinery in the process of its use are described in detail by L. Semenenko *et al.* (2021). Based on a systematic analysis of methods for increasing the durability and wear resistance of the working bodies of tillage machines, it is established that their increase in 90% of cases is achieved by technological methods. According to S. Mudarisov *et al.* (2019), a new area in the development of agriculture is emerging in the world, in fact, it is a new strategy based on the use of modern innovative technologies for tillage and the development of combined units for their implementation. However, the study of the process of interaction of tillage working bodies with the soil when designing new structures requires simple analytical physical and mathematical models of such a process. Thus, K. Aikins *et al.* (2023) modelled the interaction process of the most common tillage implements, such as cultivator tine, disc harrow on an elastic rack, deep ripper (chisel plough), mouldboard plough, and smooth roller. Analysing the interaction of these working bodies with the soil, the researchers paid attention to such parameters as the depth of cultivation, the degree of soil destruction, rolling resistance, energy costs, and other characteristics. They found that the efficiency of tillage significantly depends on the selection and adjustment of working bodies, as well as on soil parameters such as humidity and soil type. The results of the study helped to improve tillage technologies and reduce energy consumption, which corresponds to modern principles of sustainable agriculture. S. Cao *et al.* (2022) analysed the trajectory of the hook-shaped teeth, and determined the optimal operating parameters of the harvesting device. The results of the study showed that the optimal operating parameters of the harvesting device significantly affect the harvesting efficiency, reducing plant damage and grain loss. Analysis of the trajectory of the hook-shaped teeth determined the optimal depth of penetration of the teeth into the soil, the angle of inclination, and the speed of movement of the device. Q. Wang *et al.* (2021) proposed a new method for modelling the clutch for a pickup baler using automatic dynamic analysis of mechanical systems and the discrete element method.

The speed of movement of the strip, filling of buckets with seeds, backflow, surface areas of working bodies, clamping of grains in the gaps between the bucket and the strip, and other factors in the aggregate and relationship with the biological and mechanical-technological properties of grain seeds affect the injury of grains and their quality indicators. I. Rogovskii *et al.* (2020) investigated the modes of interaction of an elastic riser pipe of a spherical disk with the ground medium using ballast loading when the translational velocity of a mobile energy vehicle changes.

Spring teeth play a key role in the tillage process, providing the necessary flexibility and adaptability in harsh working conditions. Their certain ordered number forms rakes, which are the working bodies of balers, reapers, rakes, cultivators, and harrows. An important function of mulching the soil in the spring-autumn period, and the effective distribution of stubble in autumn after harvesting, is most effectively performed by heavy wide tooth harrows, the tools of which consist of a pivotally installed conical spring with one or two free tips (a tool with a spring tooth). The study by L. Tulaganova *et al.* (2022) is devoted to improving the resource performance of cultivator legs when using a combined strengthening method. The article presents a scheme of combined three-layer cultivator blade formation, which ensures an increase in resource indicators due to the application of powdered abrasion-resistant materials by arc surfacing. V. Bulgakov *et al.* (2018) proposed the design of a new spring-toothed harrow that has mechanical selectivity for weed germination. The study by H. Sheng *et al.* (2022) is devoted to the construction of mathematical models of the stability of the functioning of a mechanical rotor system. In the paper, a dynamic model of the rotor is developed using the theory of blade elements.

The elasticity of the material must be considered in many cases. For example, S. Pylypaka *et al.* (2020) considered the movement of a flexible strip up the surface of a horizontal cylinder. Moving along the inner surface of the cylinder, the strip is forced to take on its shape. Additional force is required to overcome the elasticity of the strip, unlike a completely flexible strip. The elastic bending of the riser of the cultivator tine is discussed by S. Pylypaka *et al.* (2023). The study considers the elastic axis of the tine when it is bent during operation. The study by V. Khopost & I. Demchuk (2023) is devoted to the search for the elastic axis of a spiral spring in a free state. A characteristic feature of these studies is that the elastic element is not divided into parts, that is, the change in the curvature of the elastic axis is natural.

The spring tooth of an agricultural tool consists of two sections: rectilinear and curvilinear in the form of a spring. The curvature of the elastic axis of these sections at the junction point changes abruptly from zero to a stable value. This circumstance does not allow considering the tooth as a single whole, in mathematical terms. All this determines the purpose of the study aimed at mathematically describing the deformation of the tooth, which consists of rectilinear and curvilinear sections, under the action of a force applied to it.

MATERIALS AND METHODS

The elastic tooth of an agricultural machine can be made either paired or not paired. Figure 1 shows the tooth variants and the solid model of the unpaired tooth, which was tested for bending in this study.

Figure 1. Spring tooth options and its solid-state model
Note: this model was adopted for bending tests under the applied force
Source: compiled by the authors

The tooth is conventionally divided into two parts: rectilinear and curvilinear in the form of a spring. Given that the spring pitch is a small value compared to other dimensions, a simplification was adopted, according to which it is equal to zero. From the design of the teeth, it can be seen that the end of the curvilinear part of the tooth is rigidly attached to the shaft on which the rake is located, that is, it is cantilevered.

An elastic tooth with the dimensions given on the Lidius website (n.d.) was used for consideration. Figure 2 shows a paired tooth for a heavy harrow in two projections with the required dimensions.

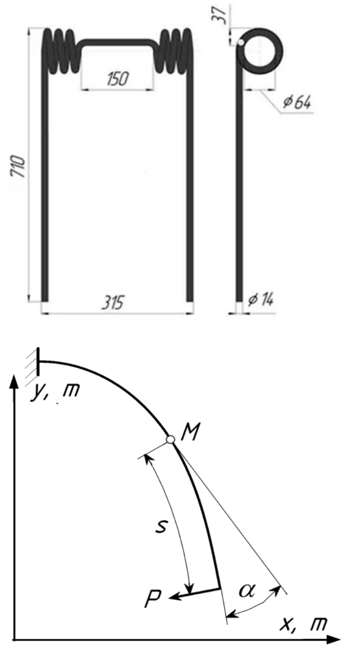

Figure 2. Spring tooth
Note: the dimensions required to calculate tooth bending under the applied force are provided
Source: Lidius (n.d.)

The radius of its elastic axis can be found from a given internal diameter of the spring: $r_o = 0.039\ m$. Value k_o can be defined as the inverse of r_o: $k_o = 25.64\ m^{-1}$. The cross-section of the tooth is a circle. Its moment of inertia I is known: $I = \pi r^4/4$, where $r = 0.007\ m$ according to Figure 2. The tooth material is spring steel. Its Young's modulus is also known: $E = 2.2 \cdot 10^{11}\ N/m^2$. Hence, the stiffness of the tooth is: $EI = 414.9\ N \cdot m^2$.

The research methods were based on the theory of material resistance and differential geometry of curves. The first part was based on the position of the resistance of materials, in relation to which the relationship between the curvature of the elastic axis of the beam, the moment applied to it, the shape of the cross-section of the beam and the Young's modulus of the material from which it is made are established. This relationship has a form:

$$k(s) = \frac{M(s)}{EI}, \qquad (1)$$

where $k = k(s)$ – dependence of the curvature k of the elastic axis depends on the length of its arc s; $M(s)$ – dependence of the applied moment M to the beam from the arc length of its elastic axis; EI – stiffness of the beam, which is the product of the Young's modulus E at the moment of inertia I of the beam cross-section. In this case, the cross-section is constant along the entire length of the beam, hence the stiffness EI is also a constant value.

The second part was based on the position of differential geometry, which establishes a relationship between the natural equation of the elastic axis, that is, the dependence of its curvature on the arc length, and parametric equations. Deformation of a cantilevered rectilinear beam under the action of an applied force *P* is shown in Figure 3.

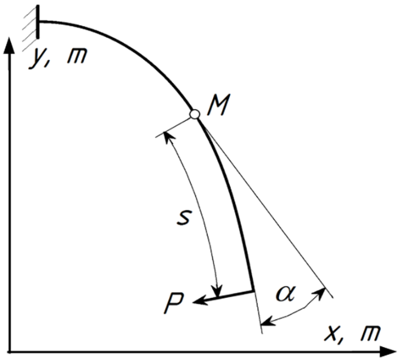

Figure 3. Graphic illustration for determining the curvature of the elastic axis of a beam at the current point
Note: under the action of the applied force *P* cantilevered rectilinear beam has acquired a curvilinear shape. *P* – applied force; *M* – current point on the elastic axis of the beam; *s* – length of the elastic axis from the point of application of the force to the current point; α – angle by which the elastic axis deviated, starting from the current point *M,* due to the action of force *P*
Source: compiled by the authors

By drawing a tangent at the current point *M* and at the point where the force *P* is applied, an angle α will be created between them. The value of this angle over the arc length interval *s* characterises the degree of deviation of the curve from the rectilinear shape, and their ratio is the average curvature. When moving to the boundary, the exact value of the curvature can be obtained at the current point:

$$k(s) = \frac{d\alpha}{ds}. \quad (2)$$

Given (2), expression (1) can be rewritten as follows:

$$\frac{d\alpha}{ds} = \frac{M(s)}{EI}. \quad (3)$$

If the dependency α=α**(s)** is known, which is found by integrating expression (3), then the parametric equations of the curve have the form of the following integrals:

$$x = \int \cos\alpha(s)\, ds;\ y = \int \sin\alpha(s)\, ds. \quad (4)$$

Even for the simplest cases of rod bending, expressions (4) cannot be integrated and require the use of numerical methods. With the advent of computer technologies, it became possible not only to numerically integrate expressions, but also to visualise the results obtained, which for this case is the construction of curvilinear elastic axes when bending rods.

RESULTS AND DISCUSSION

First, it is advisable to consider the cantilevered rectilinear part of the tooth. Under the action of the applied force *P*, which can be the reaction of the soil, it will take on a curvilinear shape (Fig. 3). Moment of force M_1 is determined from the dependence:

$$M_1 = Ps_1, \quad (5)$$

where s_1 – length of the elastic axis, which starts from the point of application of the force *P.*

After substituting (5) into (3) and integrating:

$$\alpha = \frac{Ps_1^2}{2EI}. \quad (6)$$

Further substitution of expression (6) in (4) provides parametric equations of the elastic axis, for the construction of which it is necessary to apply numerical integration methods. Arc length s_1 changes from zero to the value of the length of the rectilinear part of the tooth.

The second part of the tooth is a cantilevered spring. In the free state, it has an initial constant curvature k_o, which is the inverse of the radius r_o of the circle, the spring axis. The same force *P* will be applied to it, which is called a tracking force because it remains perpendicular to the elastic axis when it is deformed. Moment of force M_2 can increase or decrease the curvature of the elastic axis of the spring depending on how the force *P* is directed – on "twisting" or "untwisting" the spring. With this in mind, it can be written as:

$$\frac{d\alpha}{ds} = k_0 \pm \frac{M_2(s_2)}{EI} = k_0 \pm \frac{Ps_2}{EI}, \quad (7)$$

where s_2 – length of the elastic spring axis.

Equation (7) describes the shape of the elastic axis of a spring, provided that the force *P* is applied at the opposite end from fitting, that is, without considering the additional action from the rectilinear part of the tooth. This action is determined by the numerical value of the moment M_1, which can be found by equation (5). Therefore, the equation of the elastic axis of the curvilinear part of the tooth, considering the transfer of force from the rectilinear section, is written:

$$\frac{d\alpha}{ds} = k_0 \pm \frac{Ps_2 + M_1}{EI}. \quad (8)$$

Under the transfer of force from a rectilinear section of the tooth, it should be understood that this section was such before the deformation. After the applied moment, it also becomes curvilinear. After integrating expression (8):

$$\alpha = k_0 s_2 \pm \frac{Ps_2^2 + 2M_1 s_2}{2EI}. \quad (9)$$

Further application of equations (4) to construct the elastic spring axis after deformation requires numerical integration.

Firstly, it is necessary to consider the deformation of the rectilinear part of the tooth under the action of the applied force *P*. The length of its axis according to Figure 3 is $s_1 = 0.664\ m$. The force *P = 200 N was assumed.* Equation (6) determines the angle α between tangents at the start and end points of the elastic axis: α = *0.11 rad = 6.3°*. The shape of the elastic axis can be found by numerically integrating equations (4) by substituting expression (6) in them and changing s_1 within $s_1 = 0 \ldots 0.664$. The integration result is shown in Figure 4a. It shows that with increasing distance s_1, the curvature of the axis increases as the moment M_1 increases.

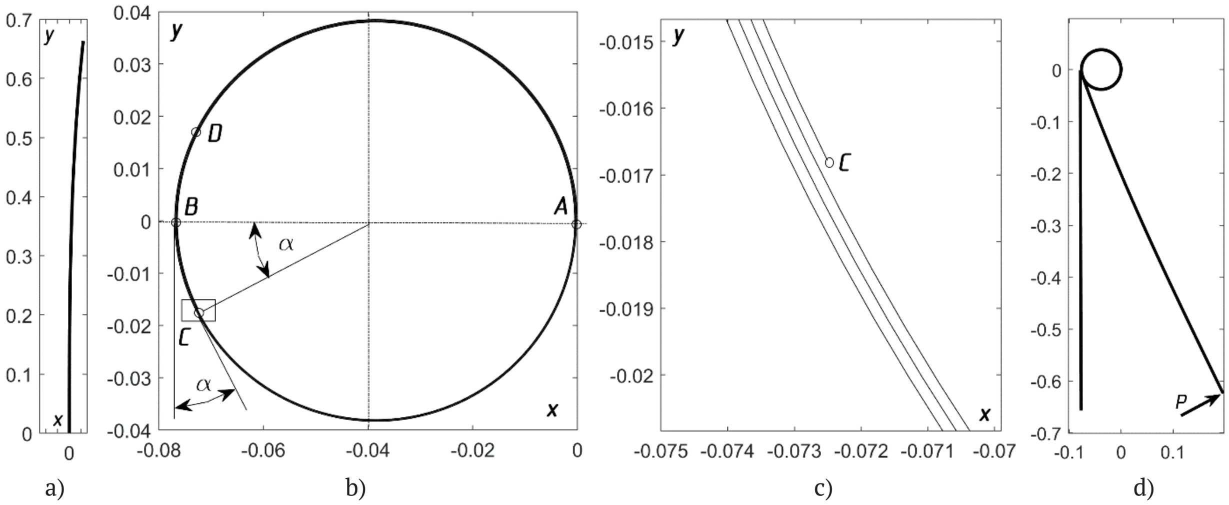

Figure 4. Graphic illustrations for finding the shape of the elastic axis of individual parts of the tooth and the tooth as a whole after the applied force of *P=200 N*
Note: a) elastic axis of the rectilinear section of the tooth after deformation; b) elastic axis of the curvilinear section of the tooth after deformation; c) enlarged fragment in the vicinity of the point *C*; d) elastic axis of the tooth
Source: developed by the authors based on research

The next step is to consider the deformation of the curvilinear part of the tooth in the form of a spring. The length of the elastic axis of 3.5 turns in the free state at $r_o = 0.039$ is $s_2 = 0.858$. The angle of rotation of the tangent corresponding to 3.5 turns is *22 rad*. After "twisting" the spring (i.e., with the "+" sign in equation (9), the new angle value is *22.45 rad*. The difference between these values: α = *0.45 rad = 25.8°*. After numerical integration of equations (4), after substituting the dependence (9) in them, it is possible to obtain the shape of the elastic axis of the spring after "twisting" due to the action of the applied force *P* (Fig. 4b). At first glance, it may seem that no changes have occurred. However, the twisting angle increased by α, that is, the free end of the spring has moved from point *B* to point *C*. Point *A* is the point of cantilever attachment of the spring to the harrow body. Figure 4b shows a repeatedly enlarged fragment that is located near point *C* highlighted with a rectangle. It shows that the elastic axis of the spring has turned from a circle into a spiral with a very high density of twists, which can be approximately determined from the scale of the enlarged fragment. To get an image of the elastic axis of the tooth as a whole after deformation by its force *P = 200 N*, it is necessary to combine the two parts shown in Figure 4a and 4b. This connection should occur at the top point of the first tooth element (Fig. 4a) with the second element (Fig. 4b) at the point *C* along the first order of smoothness, i.e., with a common tangent to both curves at this point. Based on known coordinates and angles α, this connection is shown in Figure 4d. It shows the spring tooth before and after deformation. From the obtained images, it can be concluded that the deviation of the tooth from its original position in the free state largely depends on the curvilinear part, that is, on the spring and the number of its turns. The total deviation for the given example in angular measurement is *25.8° + 6.3° = 32.1°*.

Reversing the direction of the force *P* to the opposite direction will not change the shape of the elastic axis of the straight part of the tooth, but will simply change its curvature to the opposite direction. In the curvilinear part, instead of "twisting", there will be "untwisting". Equation (9) with the "-" sign gives the value *21.55 rad.* The difference is *21.55 – 22 = -0.45 rad = -25.8°*. Thus, "twisting" and "under-twisting" occurs at the same angle. For the case of "under-twisting" in Figure 4b corresponds to the point *D*. In general, for both the first and second parts of the tooth, the law of angle growth α it is linear with different proportionality coefficients. The incremental angle α is denoted by α_1 for the rectilinear part of the tooth and by α_2 – for a curvilinear part:

$$\alpha_1 = \frac{s_1^2}{2EI}P;\ \ \alpha_2 = \pm\frac{s_2^2 + 2s_1s_2}{2EI}P. \tag{10}$$

The graph of the angle α growth depending on the amount of applied force *P* is shown in Figure 5.

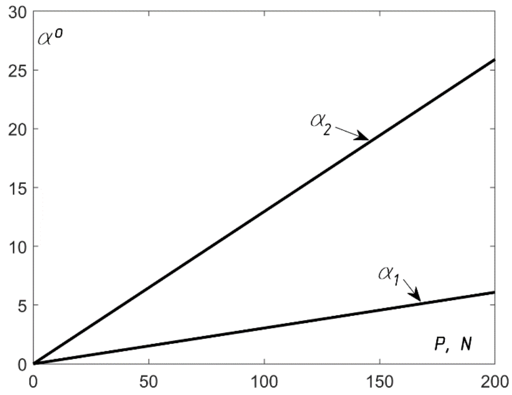

Figure 5. Graph of dependence of angle α on force *P*
Notes: for rectilinear (α_1) and curvilinear (α_2) areas of the tooth
Source: developed by the author based on research

From the graph in Figure 5 it can be seen that the intensity of angle growth α on a curvilinear section is several times higher than on a rectilinear one. From the second equation (10), it can be seen that its value largely depends on the length s_2 of the elastic axis of a curvilinear section, i.e., a spring. As its length decreases, the angle α_2 decreases. However, it is possible to reduce the stiffness proportionally by reducing the cross-sectional diameter of the tooth. This can achieve virtually the same effect, i.e., for a given force *P*, the angle α_2 will remain the same. As the stiffness decreases, the angle α_1 will increase according to the first equation (10). It is necessary to reduce the length of the spring by one twist. Then its length will be $s_2=0.61\ m$. Next, substitute the expression of the moment of inertia in the second equation (10) $I=\pi r^4/4$ and solve it with respect to *r*:

$$r = \sqrt[4]{\frac{2Ps_2(2s_1+s_2)}{\pi\alpha_2 E}}. \qquad (11)$$

All previous values of the values included in expression (11) must be taken into account, and the new value $s_2=0.61\ m$. After substituting in (11) and calculating, it is possible to obtain: $r=0.0062\ m$. Instead of the previous wire diameter *14 mm* to make a tooth, it requires *12.4 mm*. Next, the first equation (10) determines the value of the angle α_1 with a new stiffness value: $\alpha_1=9.9°$. Thus, the deformation of the rectilinear part of the tooth increased slightly – increased from $\alpha_1=6.3°$ to $\alpha_1=9.9°$.

In the future, the elastic axis calculations were performed according to the previous algorithm for new values $s_2=0.61\ m$ and $EI=255.3\ N\cdot m^2$. Figure 6 shows the same images as in Figure 4, but with the new data, i.e., the tooth diameter is *12.4 mm* and the spring has 2.5 twists instead of 3.5. For clarity, an image of the tooth and its elements with preliminary data is shown next to dashed lines. In both cases, the applied force is the same: *P=200 N*. The figures show that the differences between the shape of the tooth for the first and second cases are not significant. Figure 6c shows that with fewer twists, the distance between them increases, and the diameter of the shaft on which they should be fitted should be smaller. However, this difference can be seen under multiple magnifications. However, this must be considered, because the lack of clearance between the shaft and the spring in the free state will cause the spring to not work on twisting.

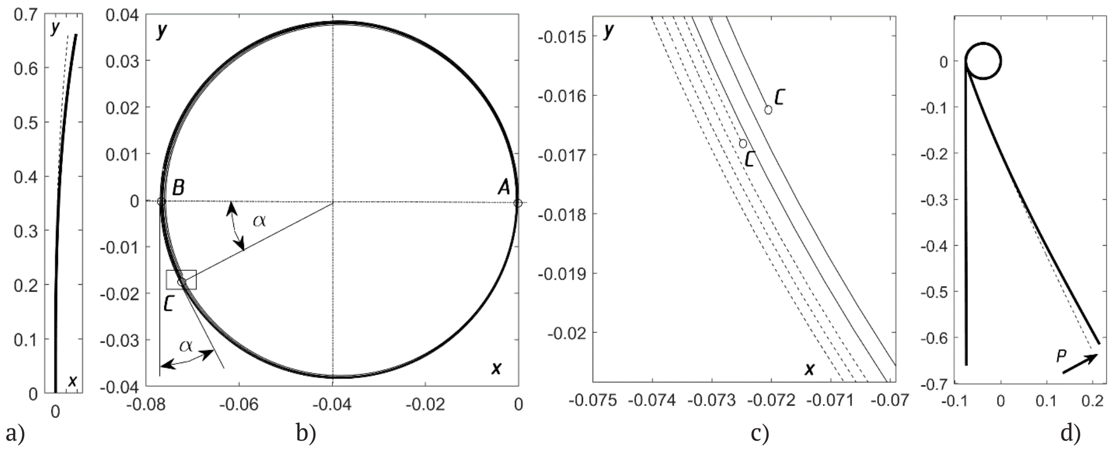

Figure 6. Comparative images of elastic axes of two tooth variants after applied force *P=200 N*
Notes: a) elastic axis of the rectilinear section of the tooth after deformation; b) elastic axis of the curvilinear section of the tooth after deformation; c) enlarged fragment in the vicinity of the point *C*, highlighted by a rectangle; d) elastic axis of the tooth
Source: developed by the author based on research

This study showed that with a slight decrease in the length of the curvilinear part (spring) and a corresponding decrease in the thickness of the tooth, it is possible to achieve almost the same power characteristics of the tooth, namely, the amount of movement of its free end under the action of a given force. In this case, the curvature of the elastic axis of the rectilinear section of the tooth is insignificant. This reduces the consumption of material for its manufacture and the weight of the finished product.

The studies by L. Babitskiy *et al.* (2020) and Ch. Li *et al.* (2021) are devoted to the investigation of the process of wear of the blades of cultivator tine blades under various types of strengthening and their influence on changes in the traction resistance of a tillage unit. The papers present dependences for predicting changes in the thickness of cutting edges reinforced with various options, on the linear wear of the blade in width and angle of its sharpening. Increasing the wear resistance of the working bodies can also be implemented by using the latest structural materials (Antoszewski *et al.*, 2017) and methods of surface nanostructuring of working surfaces (Gaponova *et al.*, 2019; Martsynkovskyy *et al.*, 2020).

B. Antoszewski *et al.* (2017) investigated the use of a UV laser with picosecond pulses to create microstructures on the surface of elastomeric plastics, which opens up new prospects in the field of material modification. This approach allows creating precise and detailed microstructures that can be used to improve the adhesion, hydrophobicity, antibacterial properties, and other characteristics of materials. The use of picosecond pulses reduces the thermal effect on the material, which ensures accuracy and control in the formation of microstructures, and also reduces the risk of deformation or damage to elastomeric plastics.

V. Martsynkovskyy *et al.* (2020) developed technological solutions to protect the contact surfaces of the coupling from fretting wear, which opens up opportunities to ensure long-term and reliable operation of mechanical connections. These technological innovations aim to prevent or minimise the impact of fretting wear on contact surfaces by applying appropriate coatings, surface treatment, or the use of special materials. This allows increasing the efficiency and service life of mechanical connections and reducing the cost of their maintenance and repair. O. Gaponova *et al.* (2019) evaluated the quality parameters of an aluminised coating produced by the electric spark alloying method, which is an important stage in the research and production process. This method allows influencing the properties of the material by injecting aluminium into the surface of the part by spark discharge. Evaluation of the quality of an aluminised coating includes analysis of parameters such as coating thickness, structure, adhesion to the substrate, microhardness, and other characteristics. Using this method improves the quality and functionality of products with an aluminised coating and ensures high quality of production.

The problem of determining deflections of beams under the action of an applied force is common in construction mechanics. However, there the deflections in comparison with the length of the beams are insignificant, that is, the curvature of the elastic axis of the beam is close to zero, that is, to a straight line. The expression of the curvature of the axis includes the first and second derivatives of its equations. Due to the fact that the deflections of the beams are small, respectively, the curvature of the axis is small, in the construction industry it is customary to neglect the second derivative, since it does not matter significantly. The problem of determining deflections is greatly simplified, that is, there is a transition from the theory of nonlinear bending to linear. If this simplification gives practical results for small deflections, then this approach is not acceptable for large deflections.

In the case of significant deflections of the rods, a nonlinear bending theory is applied. The simplest case of bending a cantilevered rod occurs when it has a rectilinear elastic axis in the free state. The study by V. Khopost & I. Demchuk (2023) considered the bending of the rod, which is the riser of a cultivator tine. The curvature of its elastic axis in the free state is constant, that is, the riser of the cultivator tine is outlined in an arc of a circle. A force is applied to the attachment point of the tine, the moment from which "spins" the riser, that is, reduces its curvature. If the initial curvature value is equated to zero, then the dependences obtained in this study describe the bending of a rectilinear rod. What is common to the current study is that the applied force changes during operation, since the working elements in the form of elastic elements interact with the soil environment, which resists variable values.

An important practical task is to determine the vibrations of elastic rods of considerable length in deep drilling structures. In them, the role of elastic rods is played by drill columns, the length of which can reach tens of meters, as noted by O. Vlasiy *et al.* (2017), M. Vasilenko *et al.* (2021). The researchers suggest that optimising the shape and size of the protective ring, which increases its thickness, is important for improving the stability of aluminium drill pipes. This optimisation is aimed at reducing the risk of damage and wear to pipes during drilling. By improving the design of the protective ring, it is possible to provide more effective protection of pipes from external influences such as friction, shock, and other mechanical loads. This approach helps to increase the service life of drill pipes and reduce the cost of their maintenance and replacement. For example, T. Wang & J. Ye (2023) consider the oscillation of a rod with a length of 100 metres. In this case, the solution to the problem is reduced to finding the functions that most accurately describe the shape of the bend line. Such a function can be polynomial. The study notes that for rods of small length, a full-scale function of the 6^{th} order can be used. However, for rods of considerable length, this function can give a significant error. Therefore, in such cases, the bend line should be considered as a spline divided into several spline functions.

An analytical description of the problem of nonlinear rod bending is given in the paper by Q. Wang *et al.* (2021). The description is based on the dependence of the curvature of the elastic axis on the applied moment and stiffness, i.e., equation (1). However, if in the current study the arc length of the elastic axis, which is natural, is taken as an independent variable, then in this paper such a variable is the coordinate "x". Accordingly, the curvature of the elastic axis is determined by the well-known equation, if the axis is defined by the dependence $y = y(x)$. Further transformations lead to a single parametric form and are expressed in terms of Jacobi elliptic functions. This approach is cumbersome and complex. The study considers three cases of rod loading: 1) bending of the rod occurs as a result of the action of a constant force in the direction, when its vector moves parallel to itself during deformation of the elastic axis; 2) bending of the rod occurs by a tracking force. In this case, the force vector retains a constant angle to the tangent of the elastic axis at the point of its application. In a special case, this angle can be a straight line. As for the conducted studies, the calculations consider the tracking force, which is perpendicular to the elastic axis at the point of its application. In general, an elastic tooth is affected by the resistance force of the soil medium, which can be decomposed into two components: perpendicular to the tooth axis and tangent to it. The ratio of these forces is of no interest, since the purpose of this study is to establish an analytical description of the deformation of a spring tooth depending on the magnitude of the applied force, which has certain limits; 3) the general case of bending is considered, when the force vector changes its direction according to a certain law, i.e., it does not follow the rotation of the elastic axis of the rod.

Characteristic of these papers is that the curvature of the rod, which can be constant or variable, is described by one equation. A characteristic difference of this study is that the elastic axis of the existing spring tooth shape cannot be described by one equation, since there is a point at which the curvature changes abruptly from zero to a certain constant value. This point is the transition from a rectilinear shape of an elastic tooth to a curvilinear one in the form of a spring. To solve such a problem, which is fundamentally different from the ones considered, it was proposed to conditionally divide the elastic axis of the tooth into two parts: curvilinear and rectilinear. When considering the deformation of a rectilinear part, it was assumed that the point of cantilever fitting is the point of interface with the curvilinear part. The expression of the change in angle α (Fig. 3) was found according to equation (6). Numerical integration of parametric equations (4) revealed a curvilinear elastic tooth axis that was rectilinear before the action of the force.

When considering the deformation of the curvilinear part of an elastic tooth, the point of cantilever attachment is the place where the tooth is attached to the unit frame. The initial curvature of the elastic axis is constant, and with this in mind, the change in the angle α was calculated using equation (9). A special feature of this equation is that, firstly, it considers the deformation of the elastic axis during "twisting" – the "+" sign, or when "untwisting" – the "-" sign. Secondly, the transfer of force from the area of the tooth that is in contact with the soil is considered. If for some reason the ground resistance force is zero, this part of the tooth becomes rectilinear and does not transmit any force to the curvilinear section of the tooth. The final stage is the connection of two sections of the tooth after deformation into a single whole along the common tangent at the point of their interface.

CONCLUSIONS

To solve the problem of mathematical description, the tooth was conventionally divided into two sections: rectilinear and curvilinear in the form of a spring. These two sections were considered separately. For this purpose, the well-known statement of the theory of material resistance on the dependence of the curvature of the elastic axis on the applied moment was applied. First, the deformation of the rectilinear section of the tooth was determined, and then the curvilinear one. However, the effect of the first section on the second was considered, since they are rigidly connected to each other, that is, they form a single whole. For both sections, it was necessary to apply numerical integration methods to find the shape of the elastic axis after their deformation. After that, both sections of the elastic axis were connected into a single whole. This approach provided an idea of the deviation of the tooth from its original position under the action of an applied force to its free end.

The proposed approach allows modifying the spring tooth, while maintaining its functionality. In particular, a variant is considered when the tooth diameter can be reduced and at the same time the length of its curvilinear section can be reduced, provided that the deviation value of its free end is commensurate with the unmodified version under the action of the same force. At the same time, it is necessary to consider the side effect, which is that the curvature of both parts of the tooth increases. This growth is characterised by the value of the angle between the tangents at the initial and final points of the elastic axis of the tooth. In the course of the study, it was found that the growth rate of this angle is much higher for the curvilinear section of the tooth. In practice, this means that the rectilinear area of the tooth has little effect on its deviation from its original position. The determining value of this deviation is the deformation of the curvilinear section of the tooth in the form of a spring. This leads to a smooth reduction in the radius of the elastic axis of the spring, which must be considered to ensure the necessary clearance between the spring and the shaft on which it is attached. If the tooth works to "untwist" the spring, that is, the applied force is directed in the opposite direction, there is a smooth increase in the radius of the elastic axis of the spring and the size of the gap or its absence does not matter.

Future studies should open up opportunities for developing new technological solutions in the field of agricultural machinery aimed at improving productivity, reducing wear, and improving consistency with other machine elements.

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CONFLICT OF INTEREST

None.

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Сергій Федорович Пилипака

Доктор технічних наук, професор
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0002-1496-4615

Тетяна Миколаївна Воліна

Кандидат технічних наук, доцент
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0001-8610-2208

В'ячеслав Іванович Хропост

Аспірант
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0001-9363-3955

Олена Григорівна Козлова

Кандидат педагогічних наук, професор
Сумський державний педагогічний університет імені А. С. Макаренка
40002, вул. Роменська, 87, м. Суми, Україна
https://orcid.org/0000-0003-1626-5188

Олександр Володимирович Таценко

Старший викладач
Сумський національний аграрний університет
40021, вул. Г. Кондратьєва, 160, м. Суми, Україна
https://orcid.org/0000-0003-1762-8219

Дослідження деформації пружинного зуба сільськогосподарських знарядь від дії прикладеної до нього сили

Анотація. Актуальність дослідження полягає в необхідності вивчення залежності сили, яку застосовують до пружинного зуба, від його параметрів, що є важливою задачею у зв'язку з широким застосуванням пружинних зубів у сільськогосподарських машинах, таких як прес-підбирачі, жатки, граблі та інші. Метою роботи є встановлення аналітичного опису деформації пружинного зуба в залежності від величини прикладеної сили. Для цього було застосовано теорію згину стержнів із опору матеріалів, при цьому вона не спрощується, як це прийнято в будівельній справі, де прогин балки в порівнянні з її довжиною є величина мала. За основу розрахунку взято відому залежність кривини пружної осі бруса (зуба) від прикладеного моменту і жорсткості його поперечного перерізу. В статті розглянуто консольно защемлений зуб, який в точці защемлення представляє собою пружину з декількома витками з подальшим плавним переходом до прямолінійної форми. Зуб по довжині розбивається на дві частини: криволінійну і прямолінійну. Розрахунок деформації, тобто знаходження форми пружної осі після дії прикладеної сили здійснюється для обох частин окремо. Необхідність у такому підході продиктована тим, що кривина пружної осі зуба у вільному стані змінюється стрибкоподібно від сталого значення криволінійної частини до нульового значення прямолінійної частини. Основним результатом дослідження є знаходження форми пружної осі окремих частин зуба під дією прикладеної сили та їх об'єднання в одне ціле. Це дає можливість визначити величину переміщення вільного кінця зуба в залежності від величини прикладеної до нього сили. Застосування отриманих даних може допомогти в розробці більш ефективних та продуктивних сільськогосподарських інструментів, а також підвищити їхню довговічність та ефективність при взаємодії з ґрунтом

Ключові слова: кривина; пружна вісь; прикладена сила; момент